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Separation Axioms via α^m-Kernel Set associated with α^m-Closed Set

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Abstract: In this paper, we introduce a new class of sets called α^m-kernel set and study their basic properties in topological spaces. We introduce and investigate some separation axioms by using α^m-kernel set and the α^m-closed set. Further, we also introduce topological α^m-kr-space.
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Keywords: α^m-closed set, α^m-kernel set, α^m-R_i-space, $i = 0,1$ and α^m-T_i-space, $i = 0,1,2$.

1. Introduction

In 1943, N. A. Shanin [9] offered a new separation axiom called R_0-space. In the same year, J. W. T. Youngs [5] introduced the first separation axiom between T_0 and T_1 spaces. In 1965, O. Njastad [10] introduced the concept of α-open sets in topological spaces. In 1970, N. Levine [8] first considered the concept of generalized closed sets were defined and investigated. In 2012, L. A. Al-Swidi and B. Mohammed [6] introduced the separation axioms via kernel set in topological spaces. In 2014, M. Mathew and R. Parimelazhagan [7] introduced the concept of α^m-closed sets in topological spaces. The purpose of this paper is to introduce the concept α^m-kernel set and to study some of its properties in topological spaces. We also investigate some of the properties of α^m-separation axioms like α^m-R_i-space, $i = 0,1$ and α^m-T_i-space, $i = 0,1,2$. Also in this paper we introduce topological α^m-kr-space iff α^m-kernel of a subset A of X is an α^m-open set. Via this kind of a topological space, we give a new characterization of separation axioms lying between α^m-T_i-space, $i = 0,1,2$.

2. Preliminaries

Throughout this paper (X,τ) or simply X will always denote a topological space. For a subset A of a topological space (X,τ), $int(A)$, $cl(A)$ and A^c represents the interior of A, the closure of A and the complement of A in X respectively.

Definition 2.1:[3] The intersection of all open subsets of a topological space (X,τ) containing A is called the kernel of A (briefly $ker(A)$), this means that $ker(A) = \bigcap\{G \in \tau : A \subseteq G\}$.

Definition 2.2:[4] Let (X,τ) be a topological space, a point x is an adherent point of $A \subseteq X$ if and only if for each $U \in \tau, x \in U$ then $A \cap U \setminus \{x\} \neq \phi$.

Definition 2.3:[10] A subset A of a topological space (X,τ) is called alpha open set (briefly α-open set) if $A \subseteq int(cl(int(A)))$ and alpha closed set (briefly α-closed set) if $cl(int(cl(A))) \subseteq A$. The α-closure of a set A of (X,τ) is the intersection of all α-closed sets that contain A and is denoted by $\alpha cl(A)$.

Definition 2.4: A subset A of a topological space (X,τ) is called:
(i) generalized closed set (briefly g-closed set) [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(ii) alpha generalized closed set (briefly αg-closed set) [2] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(iii) generalized alpha closed set (briefly gα-closed set) [1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.

Remark 2.5:[8,10] In a topological space (X,τ), the following hold and the converse of each statement is not true:
(i) Every closed set is α-closed.
(ii) Every closed set is g-closed.

Remark 2.6:[1,2] In a topological space (X,τ), the following hold and the converse of each statement is not true:
(i) Every g-closed set is αg-closed.
(ii) Every α-closed set is gα-closed.
(iii) Every gα-closed set is αg-closed.

Definition 2.7:[7] A subset A of a topological space (X,τ) is called α^m-closed set if $int(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is α-open. The complement of α^m-closed set in X is α^m-open in X, the family of all α^m-open (α^m-closed) sets of a topological space (X,τ) is denoted by α^m-$O(X)$ (α^m-$C(X)$).

Definition 2.8:[7] The intersection of all α^m-closed sets in X containing A is called α^m-closure of A and is denoted by α^m-$cl(A)$, α^m-$cl(A) = \bigcap\{B : A \subseteq B, B$ is α^m-closed $\}$.

Remark 2.9:[7] In a topological space (X,τ), the following hold and the converse of each statement is not true:
(i) Every closed set is α^m-closed.
(ii) Every α^m-closed set is α-closed.
(iii) Every α^m-closed set is αg-closed.
(iv) Every α^m-closed set is gα-closed.

Theorem 2.10:[7] A set A is α^m-closed set iff $int(cl(A)) - A$ contains no nonempty α^m-closed sets.

Theorem 2.11:[7] Let $B \subseteq Y \subseteq X$, if B is α^m-closed set relative to Y and Y is open then B is α^m-closed set in X.

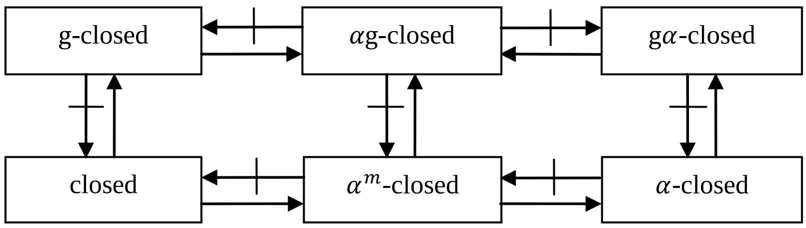
Theorem 2.12:[7] If A is α^m-closed set and $A \subseteq B \subseteq int(cl(A))$ then B is α^m-closed set.

Theorem 2.13:[7] The intersection of α^m-closed set and a closed set is α^m-closed set.

Theorem 2.14:[7] If A and B are two α^m-closed sets defined for a nonempty set X, then their intersection $A \cap B$ is α^m-closed set in X.

Remark 2.15:[7] The union of two α^m-closed sets need not be α^m-closed set.

Remark 2.16: The following are the implications of α^m-closed set and the reverse is not true.



3. α^m-Kernel and α^m-R_i-Spaces, $i = 0, 1$

Definition 3.1: The intersection of all α^m-open subset of X containing A is called the α^m-kernel of A (briefly α^m-$ker(A)$), this means α^m-$ker(A) = \bigcap\{G \in \alpha^m\text{-}O(X) : A \subseteq G\}$.

Definition 3.2: Let x be a point of a topological space X. The α^m-kernel of x, denoted by α^m-$ker(\{x\})$ is defined to be the set α^m-$ker(\{x\}) = \bigcap\{G : G \in \alpha^m\text{-}O(X)$ and $x \in G\}$.

Lemma 3.3: Let (X,τ) be a topological space, then $y \in \alpha^m$-$ker(\{x\})$ if and only if $x \in \alpha^m$-$cl(\{y\})$ for each $x \neq y \in X$.

Proof: Suppose that $y \notin \alpha^m$-$ker(\{x\})$. Then there exists α^m-open set U containing x such that $y \notin U$. Therefore, we have $x \notin \alpha^m$-$cl(\{y\})$. The converse part can be proved in a similar way.

Definition 3.4: A set A in topological space (X,τ) is called α^m-neighborhood (briefly α^m-nhd) of a point x if there exists α^m-open set B such that $x \in B \subseteq A$.

Lemma 3.5: Let (X,τ) be a topological space and A be a subset of X. Then, α^m-$ker(A) = \{x \in X : \alpha^m$-$cl(\{x\}) \cap A \neq \phi\}$.

Proof: Let $x \in \alpha^m$-$ker(A)$ and α^m-$cl(\{x\}) \cap A = \phi$. Hence $x \notin X - \alpha^m$-$cl(\{x\})$ which is α^m-open set containing A. This is impossible, since $x \in \alpha^m$-$ker(A)$.

Consequently, α^m-$cl(\{x\}) \cap A \neq \phi$. Next, let $x \in X$ such that α^m-$cl(\{x\}) \cap A \neq \phi$ and suppose that $x \notin \alpha^m$-$ker(A)$. Then there exists α^m-open set U containing A and $x \notin U$. Let $y \in \alpha^m$-$cl(\{x\}) \cap A$. Hence, U is α^m-nhd of y which does not contain x. By this contradiction $x \in \alpha^m$-$ker(A)$ and the claim.

Definition 3.6: Let (X, τ) be a topological space. A point x is said to be:
(i) α^m-adherent point of $A \subseteq X$ if and only if for each $U \in \alpha^m$-$O(X)$, $x \in U$ then $A \cap U \setminus \{x\} \neq \phi$.
(ii) α^m-kernelled point of $A \subseteq X$ (briefly $x \in \alpha^m$-$ker(A)$) if and only if for each F α^m-closed set contains x then $F \cap A \neq \phi$.
(iii) boundary α^m-kernelled point of A (briefly $x \in \alpha^m$-$ker_{bd}(A)$) if and only if for each α^m-closed set F contains x then $F \cap A \neq \phi$ and $F \cap A^c \neq \phi$.
(iv) derived α^m-kernelled point of A (briefly $x \in \alpha^m$-$ker_{dr}(A)$) if and only if for each F α^m-closed set contains x then $A \cap F/\{x\} \neq \phi$.

Definition 3.7: By definition (3.6)(ii), we have the following: For every two distinct point x and y of X, α^m-$ker(\{x\}) = \{y : x \in F_y, F_y^c \in \alpha^m\text{-}O(X)\}$.

Theorem 3.8: Let (X, τ) be a topological space and $x \neq y \in X$. Then x is α^m-kernelled point of $\{y\}$ if and only if y is an α^m-adherent point of $\{x\}$.

Proof: Let x be an α^m-kernelled point of $\{y\}$. Then for every α^m-closed set F such that $x \in F$ implies $y \in F$, then $y \in \cap\{F : x \in F\}$, this means $y \in \alpha^m$-$cl(\{x\})$. Thus y is an α^m-adherent point of $\{x\}$.
Conversely, let y be an α^m-adherent point of $\{x\}$. Then for every α^m-open set U such that $y \in U$ implies $x \in U$, then $x \in \cap\{U : y \in U\}$, this means $x \in \alpha^m$-$ker(\{y\})$. Thus, x is α^m-kernelled point of $\{y\}$.

Theorem 3.9: Let (X, τ) be a topological space and $A \subseteq X$ and let α^m-$ker_{dr}(A)$ be the set of all derived α^m-kernelled derived points of A, then α^m-$ker(A) = A \cup \alpha^m$-$ker_{dr}(A)$.

Proof: Let $x \in A \cup \alpha^m$-$ker_{dr}(A)$ and if $x \in \alpha^m$-$ker_{dr}(A)$, then for every α^m-closed set F intersects A (in a point different from x). Therefore, $x \in \alpha^m$-$ker(\{x\})$. Hence, α^m-$ker_{dr}(A) \subseteq \alpha^m$-$ker(A)$, it follows that $A \cup \alpha^m$-$ker_{dr}(A) \subseteq \alpha^m$-$ker(A)$. To demonstrate the reverse inclusion, we consider x be a point of α^m-$ker(A)$. If $x \in A$, then $x \in A \cup \alpha^m$-$ker_{dr}(A)$. Suppose that $x \notin A$. Since $x \in \alpha^m$-$ker(A)$, then for every α^m-closed set F containing x implies $F \cap A \neq \phi$, this means $A \cap F/\{x\} \neq \phi$. Then, $x \in \alpha^m$-$ker_{dr}(A)$, so that $x \in A \cup \alpha^m$-$ker_{dr}(A)$. Hence, α^m-$ker(A) \subseteq A \cup \alpha^m$-$ker_{dr}(A)$. Thus, α^m-$ker(A) = A \cup \alpha^m$-$ker_{dr}(A)$.

Theorem 3.10: Let (X, τ) be a topological space and $A \subseteq X$ and let α^m-$ker_{bd}(A)$ be the set of all boundary α^m-kernelled points of A, then α^m-$ker(A) = A \cup \alpha^m$-$ker_{bd}(A)$.

Proof: Let $x \in A \cup \alpha^m$-$ker_{bd}(A)$ and if $x \in \alpha^m$-$ker_{bd}(A)$, then for every α^m-closed set F intersects A, therefore $x \in \alpha^m$-$ker(\{x\})$. Hence, α^m-$ker_{bd}(A) \subseteq \alpha^m$-$ker(A)$, it follows that $A \cup \alpha^m$-$ker_{bd}(A) \subseteq \alpha^m$-$ker(A)$. To demonstrate the reverse inclusion, we consider x be a point of α^m-$ker(A)$. If $x \in A$, then $x \in A \cup \alpha^m$-$ker_{bd}(A)$. Suppose that $x \notin A$, implies $x \in A^c$. Since $x \in \alpha^m$-$ker(A)$, then for every α^m-closed set F containing x implies $F \cap A \neq \phi$ and $F \cap A^c \neq \phi$. Then $x \in \alpha^m$-$ker_{bd}(A)$, so that $x \in A \cup \alpha^m$-$ker_{bd}(A)$. Hence, α^m-$ker(A) \subseteq A \cup \alpha^m$-$ker_{bd}(A)$. Thus, α^m-$ker(A) = A \cup \alpha^m$-$ker_{bd}(A)$.

Definition 3.11: In a topological space (X, τ), a set A is said to be weakly ultra α^m-separated from B if there exists α^m-open set G such that $G \cap B = \phi$ or $A \cap \alpha^m$-$cl(B) = \phi$.

By definition (3.11), we have the following: For every two distinct points x and y of X,
(i) α^m-$cl(\{x\}) = \{x : \{y\}$ is not weakly ultra α^m-separated from $\{x\}\}$.
(ii) α^m-$ker(\{x\}) = \{y : \{x\}$ is not weakly ultra α^m-separated from $\{y\}\}$.

Definition 3.12: A topological space (X, τ) is called α^m-R_0-space if for each α^m-open set U and $x \in U$, then α^m-$cl(\{x\}) \subseteq U$.

Definition 3.13: A topological space (X, τ) is called α^m-R_1-space if for each two distinct points x and y of X with α^m-$cl(\{x\}) \neq \alpha^m$-$cl(\{y\})$, there exist disjoint α^m-open sets U, V such that α^m-$cl(\{x\}) \subseteq U$ and α^m-$cl(\{y\}) \subseteq V$.

Theorem 3.14: Let (X, τ) be a topological space. Then (X, τ) is α^m-R_0-space if and only if α^m-$cl(\{x\}) = \alpha^m$-$ker(\{x\})$, for each $x \in X$.

Proof: Let (X,τ) be an α^m-R_0-space. If α^m-$cl(\{x\}) \neq \alpha^m$-$ker(\{x\})$, for each $x \in X$, then there exist another point $y \neq x$ such that $y \in \alpha^m$-$cl(\{x\})$ and $y \notin \alpha^m$-$ker(\{x\})$ this means there exist an U_x α^m-open set, $y \notin U_x$ implies α^m-$cl(\{x\}) \nsubseteq U_x$ this contradiction. Thus α^m-$cl(\{x\}) = \alpha^m$-$ker(\{x\})$.
Conversely, let α^m-$cl(\{x\}) = \alpha^m$-$ker(\{x\})$, for each α^m-open set $U, x \in U$, then α^m-$ker(\{x\}) = \alpha^m$-$cl(\{x\}) \subseteq U$ [by definition (3.1)]. Hence by definition (3.12), (X,τ) is α^m-R_0-space.

Theorem 3.15: A topological space (X,τ) is α^m-R_0-space if and only if for each F α^m-closed set and $x \in F$, then α^m-$ker(\{x\}) \subseteq F$.

Proof: Let for each F α^m-closed set and $x \in F$, then α^m-$ker(\{x\}) \subseteq F$ and let U be α^m-open set, $x \in U$ then for each $y \notin U$ implies $y \in U^c$ is α^m-closed set implies α^m-$ker(\{y\}) \subseteq U^c$[by assumption]. Therefore $x \notin \alpha^m$-$ker(\{y\})$ implies $y \notin \alpha^m$-$cl(\{x\})$ [by lemma (3.3)]. So α^m-$cl(\{x\}) \subseteq U$. Thus (X,τ) is α^m-R_0-space.
Conversely, let (X,τ) be an α^m-R_0-space and F be α^m-closed set and $x \in F$. Then for each $y \notin F$ implies $y \in F^c$ is α^m-open set, then α^m-$cl(\{y\}) \subseteq F^c$[since (X,τ) is α^m-R_0-space], so α^m-$ker(\{x\}) = \alpha^m$-$cl(\{x\})$. Thus, α^m-$ker(\{x\}) \subseteq F$.

Corollary 3.16: A topological space (X,τ) is α^m-R_0-space if and only if for each U α^m-open set and $x \in U$, then α^m-$cl(\alpha^m$-$ker(\{x\})) \subseteq U$.

Proof: Clearly.

Theorem 3.17: Let (X,τ) be a topological α^m-R_0-space. Then the following statements are equivalent
(i) Every α^m-kernelled point of $\{x\}$ is an α^m-adherent point of $\{x\}$.
(ii) Every α^m-adherent point of $\{x\}$ is an α^m-kernelled point of $\{x\}$.

Proof: (i) Let (X,τ) be an α^m-R_0-space. Then, for each $x \in X$, α^m-$cl(\{x\}) = \alpha^m$-$ker(\{x\})$ [by theorem (3.14)]. Thus, every α^m-kernelled point of $\{x\}$ is an α^m-adherent point of $\{x\}$.
Conversely, let every α^m-kernelled point of $\{x\}$ is an α^m-adherent point of $\{x\}$ and let F be α^m-closed set, $x \in F$. Then α^m-$ker(\{x\}) \subseteq \alpha^m$-$cl(\{x\})$, for each $x \in X$. Since α^m-$cl(\{x\}) = \bigcap\{F: F \in \alpha^m\text{-}C(X), x \in F\}$, implies α^m-$ker(\{x\}) \subseteq F$. Hence by theorem (3.15), (X,τ) is an α^m-R_0-space.
(ii) Let (X,τ) be an α^m-R_0-space. Then, for each $x \in X$, α^m-$cl(\{x\}) = \alpha^m$-$ker(\{x\})$ [by theorem (3.14)]. Thus, every α^m-adherent point of $\{x\}$ is an α^m- kernelled point of $\{x\}$.
Conversely, let every α^m-adherent point of $\{x\}$ is an α^m-kernelled point of $\{x\}$ and let U be α^m-open set and $x \in U$. Then α^m-$cl(\{x\}) \subseteq \alpha^m$-$ker(\{x\})$, for each $x \in X$. Since α^m-$ker(\{x\}) = \bigcap\{U: U \in \alpha^m\text{-}O(X), x \in U\}$, implies α^m-$cl(\{x\}) \subseteq U$. Hence by definition (3.12), (X,τ) is an α^m-R_0-space.

Theorem 3.18: Every α^m-R_1-space is α^m-R_0-space.

Proof: Let (X,τ) be an α^m-R_1-space and let U be α^m-open set, $x \in U$, then for each $y \notin U$ implies $y \in U^c$ is α^m-closed set and α^m-$cl(\{y\}) \subseteq U^c$ implies α^m-$cl(\{x\}) \neq \alpha^m$-$cl(\{y\})$. Hence by definition (3.13), α^m-$cl(\{x\}) \subseteq U$. Thus (X,τ) is α^m-R_0-space.

Theorem 3.19: A topological space (X,τ) is α^m-R_1-space if and only if for each $x \neq y \in X$ with α^m-$ker(\{x\}) \neq \alpha^m$-$ker(\{y\})$, then there exist α^m-closed sets G_1, G_2 such that α^m-$ker(\{x\}) \subseteq G_1$, α^m-$ker(\{x\}) \cap G_2 = \phi$ and α^m-$ker(\{y\}) \subseteq G_2$, α^m-$ker(\{y\}) \cap G_1 = \phi$ and $G_1 \cup G_2 = X$.

Proof: Let (X,τ) be an α^m-R_1-space. Then for each $x \neq y \in X$ with α^m-$ker(\{x\}) \neq \alpha^m$-$ker(\{y\})$. Since every α^m-R_1-space is α^m-R_0-space [by theorem (3.18)], and by theorem (3.14), α^m-$cl(\{x\}) \neq \alpha^m$-$cl(\{y\})$, then there exist α^m-open sets U_1,U_2 such that α^m-$cl(\{x\}) \subseteq U_1$ and α^m-$cl(\{y\}) \subseteq U_2$ and $U_1 \cap U_2 = \phi$ [since (X,τ) is α^m-R_1-space], then U_1^c and U_2^c are α^m-closed sets such that $U_1^c \cup U_2^c = X$. Put $G_1 = U_1^c$ and $G_2 = U_2^c$. Thus $x \in U_1 \subseteq G_2$ and $y \in U_2 \subseteq G_1$ so that α^m-$ker(\{x\}) \subseteq U_1 \subseteq G_2$ and α^m-$ker(\{y\}) \subseteq U_2 \subseteq G_1$.
Conversely, let for each $x \neq y \in X$ with α^m-$ker(\{x\}) \neq \alpha^m$-$ker(\{y\})$, there exist α^m-closed sets G_1, G_2 such that α^m-$ker(\{x\}) \subseteq G_1$, α^m-$ker(\{x\}) \cap G_2 = \phi$ and α^m-$ker(\{y\}) \subseteq G_2$, α^m-$ker(\{y\}) \cap G_1 = \phi$ and $G_1 \cup G_2 = X$, then G_1^c and G_2^c are α^m-open sets such that $G_1^c \cap G_2^c = \phi$. Put $G_1^c = U_2$ and $G_2^c = U_1$. Thus, α^m-$ker(\{x\}) \subseteq U_1$ and α^m-$ker(\{y\}) \subseteq U_2$ and $U_1 \cap U_2 = \phi$, so that $x \in U_1$ and $y \in U_2$ implies $x \notin \alpha^m$-$cl(\{y\})$ and $y \notin \alpha^m$-$cl(\{x\})$, then α^m-$cl(\{x\}) \subseteq U_1$ and α^m-$cl(\{y\}) \subseteq U_2$. Thus, (X,τ) is α^m-R_1-space.

Corollary 3.20: A topological space (X,τ) is α^m-R_1-space if and only if for each $x \neq y \in X$ with α^m-$cl(\{x\}) \neq \alpha^m$-$cl(\{y\})$ there exist disjoint α^m-open sets U,V such that α^m-$cl(\alpha^m$-$ker(\{x\})) \subseteq U$ and α^m-$cl(\alpha^m$-$ker(\{y\})) \subseteq V$.

Proof: Let (X,τ) be an α^m-R_1-space and let $x \neq y \in X$ with $\alpha^m\text{-}cl(\{x\}) \neq \alpha^m\text{-}cl(\{y\})$, then there exist disjoint α^m-open sets U,V such that $\alpha^m\text{-}cl(\{x\}) \subseteq U$ and $\alpha^m\text{-}cl(\{y\}) \subseteq V$. Also (X,τ) is α^m-R_0-space [by theorem (3.18)] implies for each $x \in X$, then $\alpha^m\text{-}cl(\{x\}) = \alpha^m\text{-}ker(\{x\})$ [by theorem (3.14)], but $\alpha^m\text{-}cl(\{x\}) = \alpha^m\text{-}cl(\alpha^m\text{-}cl(\{x\})) = \alpha^m\text{-}cl(\alpha^m\text{-}ker(\{x\}))$. Thus $\alpha^m\text{-}cl(\alpha^m\text{-}ker(\{x\})) \subseteq U$ and $\alpha^m\text{-}cl(\alpha^m\text{-}ker(\{y\})) \subseteq V$.
Conversely, let for each $x \neq y \in X$ with $\alpha^m\text{-}cl(\{x\}) \neq \alpha^m\text{-}cl(\{y\})$ there exist disjoint α^m-open sets U,V such that $\alpha^m\text{-}cl(\alpha^m\text{-}ker(\{x\})) \subseteq U$ and $\alpha^m\text{-}cl(\alpha^m\text{-}ker(\{y\})) \subseteq V$. Since $\{x\} \subseteq \alpha^m\text{-}ker(\{x\})$, then $\alpha^m\text{-}cl(\{x\}) \subseteq \alpha^m\text{-}cl(\alpha^m\text{-}ker(\{x\}))$ for each $x \in X$. So we get $\alpha^m\text{-}cl(\{x\}) \subseteq U$ and $\alpha^m\text{-}cl(\{y\}) \subseteq V$. Thus, (X,τ) is α^m-R_1-space.

4. α^m-T_i-Spaces, $i = 0, 1, 2$

Definition 4.1: Let (X,τ) be a topological space. Then X is called:
(i) α^m-T_0-space iff for each pair of distinct points in X, there exists α^m-open set in X containing one and not the other.
(ii) α^m-T_1-space iff for each pair of distinct points x and y of X, there exists α^m-open sets G,H containing x and y respectively such that $y \notin G$ and $x \notin H$.
(iii) α^m-T_2-space iff for each pair of distinct points x and y of X, there exist disjoint α^m-open sets G,H in X such that $x \in G$ and $y \in H$.

Remark 4.2: Every α^m-T_i-space is α^m-T_{i-1}-space, $i = 1,2$.

Proof: Clearly.

Theorem 4.3: A topological space (X,τ) is α^m-T_0-space if and only if either $y \notin \alpha^m\text{-}ker(\{x\})$ or $x \notin \alpha^m\text{-}ker(\{y\})$, for each $x \neq y \in X$.

Proof: Let (X,τ) be an α^m-T_0-space then for each $x \neq y \in X$, there exists α^m-open set G such that $x \in G, y \notin G$ or $x \notin G, y \in G$. Thus either $x \in G, y \notin G$ implies $y \notin \alpha^m\text{-}ker(\{x\})$ or $x \notin G, y \in G$ implies $x \notin \alpha^m\text{-}ker(\{y\})$.
Conversely, let either $y \notin \alpha^m\text{-}ker(\{x\})$ or $x \notin \alpha^m\text{-}ker(\{y\})$, for each $x \neq y \in X$. Then there exists α^m-open set G such that $x \in G, y \notin G$ or $x \notin G, y \in G$. Thus (X,τ) is α^m-T_0-space.

Theorem 4.4: A topological space (X,τ) is α^m-T_0-space if and only if either $\alpha^m\text{-}ker(\{x\})$ is weakly ultra α^m-separated from $\{y\}$ or $\alpha^m\text{-}ker(\{y\})$ is weakly ultra α^m-separated from $\{x\}$ for each $x \neq y \in X$.

Proof: Let (X,τ) be an α^m-T_0-space then for each $x \neq y \in X$, there exists α^m-open set G such that $x \in G, y \notin G$ or $x \notin G, y \in G$. Now if $x \in G, y \notin G$ implies $\alpha^m\text{-}ker(\{x\})$ is weakly ultra α^m-separated from $\{y\}$. Or if $x \notin G$, $y \in G$ implies $\alpha^m\text{-}ker(\{y\})$ is weakly ultra α^m-separated from $\{x\}$.
Conversely, let either $\alpha^m\text{-}ker(\{x\})$ be weakly ultra α^m-separated from $\{y\}$ or $\alpha^m\text{-}ker(\{y\})$ be weakly ultra α^m-separated from $\{x\}$. Then there exists α^m-open set G such that $\alpha^m\text{-}ker(\{x\}) \subseteq G$ and $y \notin G$ or $\alpha^m\text{-}ker(\{y\}) \subseteq G, x \notin G$ implies $x \in G, y \notin G$ or $x \notin G, y \in G$. Thus, (X,τ) is α^m-T_0- space.

Theorem 4.5: A topological space (X,τ) is α^m-T_0-space if and only if for each $x \neq y \in X$, either x is not α^m-kernelled point of $\{y\}$ or y is not α^m-kernelled point of $\{x\}$.

Proof: Let (X,τ) be an α^m-T_0-space. Then for each $x \neq y \in X$ there exists an α^m-open set U such that $x \in U$, $y \notin U$ (say), implies $y \in U^c$. Hence U^c is α^m-closed, then y is not α^m-kernelled point of $\{x\}$ [by definition (3.6)(ii)]. Thus either x is not α^m-kernelled point of $\{y\}$ or y is not α^m-kernelled point of $\{x\}$.
Conversely, Let for each $x \neq y \in X$, either x is not α^m-kernelled point of $\{y\}$ or y is not α^m-kernelled point of $\{x\}$. Then there exist α^m-closed set F such that $x \in F$, $F \cap \{y\} = \phi$ or $y \in F$, $F \cap \{x\} = \phi$, implies $x \notin F^c$, $y \in F^c$ or $x \in F^c$, $y \notin F^c$. Hence F^c is an α^m-open set. Thus, (X,τ) is α^m-T_0-space.

Theorem 4.6: A topological space (X,τ) is α^m-T_1-space if and only if for each $x \neq y \in X$, $\alpha^m\text{-}ker(\{x\})$ is weakly ultra α^m-separated from $\{y\}$ and $\alpha^m\text{-}ker(\{y\})$ is weakly ultra α^m-separated from $\{x\}$.

Proof: Let (X,τ) be an α^m-T_1-space then for each $x \neq y \in X$, there exist α^m-open sets U,V such that $x \in U, y \notin U$ and $x \notin V, y \in V$. Implies $\alpha^m\text{-}ker(\{x\})$ is weakly ultra α^m-separated from $\{y\}$ and $\alpha^m\text{-}ker(\{y\})$ is weakly ultra α^m-separated from $\{x\}$.
Conversely, let $\alpha^m\text{-}ker(\{x\})$ be weakly ultra α^m-separated from $\{y\}$ and $\alpha^m\text{-}ker(\{y\})$ be weakly ultra α^m-separated from $\{x\}$. Then there exist α^m-open sets U,V such that $\alpha^m\text{-}ker(\{x\}) \subseteq U, y \notin U$ and $\alpha^m\text{-}ker(\{y\}) \subseteq V, x \notin V$ implies $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$. Thus, (X,τ) is α^m-T_1-space.

Theorem 4.7: A topological space (X,τ) is α^m-T_1-space if and only if for each $x \in X$, α^m-$ker(\{x\}) = \{x\}$.

Proof: Let (X,τ) be an α^m-T_1-space and let α^m-$ker(\{x\}) \neq \{x\}$. Then α^m-$ker(\{x\})$ contains another point distinct from x say y. So $y \in \alpha^m$-$ker(\{x\})$ implies α^m-$ker(\{x\})$ is not weakly ultra α^m-separated from $\{y\}$. Hence by theorem (4.6), (X,τ) is not α^m-T_1-space this is contradiction. Thus α^m-$ker(\{x\}) = \{x\}$.
Conversely, let α^m-$ker(\{x\}) = \{x\}$, for each $x \in X$ and let (X,τ) be not α^m-T_1-space. Then by theorem (4.6), α^m-$ker(\{x\})$ is not weakly ultra α^m-separated from $\{y\}$, this means that for every α^m-open set G contains α^m-$ker(\{x\})$ then $y \in G$ implies $y \in \bigcap\{G \in \alpha^m\text{-}O(X) : x \in G\}$ implies $y \in \alpha^m$-$ker(\{x\})$, this is contradiction. Thus, (X,τ) is α^m-T_1-space.

Theorem 4.8: A topological space (X,τ) is α^m-T_1-space if and only if α^m-$ker_{dr}(\{x\}) = \phi$, for each $x \in X$.

Proof: Let (X,τ) be an α^m-T_1-space. Then for each $x \in X$, α^m-$ker(\{x\}) = \{x\}$ [by theorem (4.6)]. Since α^m-$ker_{dr}(\{x\}) = \alpha^m$-$ker(\{x\}) - \{x\}$. Thus α^m-$ker_{dr}(\{x\}) = \phi$.
Conversely, let α^m-$ker_{dr}(\{x\}) = \phi$. By theorem (3.9), α^m-$ker(\{x\}) = \{x\} \cup \alpha^m$-$ker_{dr}(\{x\})$, implies α^m-$ker(\{x\}) = \{x\}$. Hence by theorem (4.7), (X,τ) is α^m-T_1-space.

Theorem 4.9: A topological space (X,τ) is α^m-T_1-space if and only if for each $x \neq y \in X$, x is not α^m-kernelled point of $\{y\}$ and y is not α^m-kernelled point of $\{x\}$.

Proof: Let (X,τ) be an α^m-T_1-space. Then for each $x \neq y \in X$, there exist α^m-open sets U, V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$ implies $x \in V^c$, $\{y\} \cap V^c = \phi$ and $y \in U^c$, $\{x\} \cap U^c = \phi$. Hence, U^c and V^c are α^m-closed sets. Thus x is not α^m-kernelled point of $\{y\}$ and y is not α^m-kernelled point of $\{x\}$.
Conversely, let for each $x \neq y \in X$, x is not α^m-kernelled point of $\{y\}$ and y is not α^m-kernelled point of $\{x\}$. Then there exist α^m-closed sets F_1, F_2 such that $x \in F_1$, $F_1 \cap \{y\} = \phi$ and $y \in F_2$, $F_2 \cap \{x\} = \phi$, implies $x \in F_2^c$, $y \notin F_2^c$ and $y \in F_1^c$, $x \notin F_1^c$. Hence F_1^c and F_2^c are α^m-open sets. Thus, (X,τ) is α^m-T_1-space.

Theorem 4.10: A topological space (X,τ) is α^m-T_1-space if and only if for each $x \neq y \in X$, $y \notin \alpha^m$-$ker(\{x\})$ and $x \notin \alpha^m$-$ker(\{y\})$.

Proof: Let (X,τ) be an α^m-T_1-space then for each $x \neq y \in X$, there exists α^m-open sets U, V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$. Implies $y \notin \alpha^m$-$ker(\{x\})$ and $x \notin \alpha^m$-$ker(\{y\})$.
Conversely, let $y \notin \alpha^m$-$ker(\{x\})$ and $x \notin \alpha^m$-$ker(\{y\})$, for each $x \neq y \in X$. Then there exists α^m-open sets U, V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$. Thus, (X,τ) is α^m-T_1-space.

Theorem 4.11: A topological space (X,τ) is α^m-T_1-space if and only if for each $x \neq y \in X$ implies α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$.

Proof: Let (X,τ) be an α^m-T_1-space. Then α^m-$ker(\{x\}) = \{x\}$ and α^m-$ker(\{y\}) = \{y\}$ [by theorem (4.7)]. Thus, α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$.
Conversely, let for each $x \neq y \in X$ implies α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$ and let (X,τ) be not α^m-T_1-space then for each $x \neq y \in X$ implies $y \in \alpha^m$-$ker(\{x\})$ or $x \in \alpha^m$-$ker(\{y\})$ [by theorem (4.10)], then α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) \neq \phi$ this is contradiction. Thus, (X,τ) is α^m-T_1-space.

Theorem 4.12: A topological space (X,τ) is α^m-T_1-space if and only if (X,τ) is α^m-T_0-space and α^m-R_0-space.

Proof: Let (X,τ) be an α^m-T_1-space and let $x \in U$ be α^m-open set, then for each $x \neq y \in X$, α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$ [by theorem (4.11)] implies $x \notin \alpha^m$-$ker(\{y\})$ and $y \notin \alpha^m$-$ker(\{x\})$ this means α^m-$cl(\{x\}) = \{x\}$, hence α^m-$cl(\{x\}) \subseteq U$. Thus, (X,τ) is α^m-R_0-space.
Conversely, let (X,τ) be an α^m-T_0-space and α^m-R_0-space, then for each $x \neq y \in X$ there exists α^m-open set U such that $x \in U$, $y \notin U$ or $x \notin U$, $y \in U$. Say $x \in U$, $y \notin U$ since (X,τ) is α^m-R_0-space, then α^m-$cl(\{x\}) \subseteq U$, this means there exists α^m-open set V such that $y \in V$, $x \notin V$. Thus, (X,τ) is α^m-T_1-space.

Theorem 4.13: A topological space (X,τ) is α^m-T_2-space if and only if
(i) (X,τ) is α^m-T_0-space and α^m-R_1-space.
(ii) (X,τ) is α^m-T_1-space and α^m-R_1-space.

Proof: (i) Let (X,τ) be an α^m-T_2-space then it is α^m-T_0-space. Now since (X,τ) is α^m-T_2-space then for each $x \neq y \in X$, there exist disjoint α^m-open sets U, V such that $x \in U$ and $y \in V$ implies $x \notin \alpha^m$-$cl(\{y\})$ and $y \notin \alpha^m$-$cl(\{x\})$, therefore α^m-$cl(\{x\}) = \{x\} \subseteq U$ and α^m-$cl(\{y\}) = \{y\} \subseteq V$. Thus, (X,τ) is α^m-R_1-space.

Conversely, let (X,τ) be an α^m-T_0-space and α^m-R_1-space, then for each $x \neq y \in X$, there exists α^m-open set U such that $x \in U$, $y \notin U$ or $y \in U$, $x \notin U$, implies α^m-$cl(\{x\}) \neq \alpha^m$-$cl(\{y\})$, since (X,τ) is α^m-R_1-space [by assumption], then there exist disjoint α^m-open sets G, H such that $x \in G$ and $y \in H$ [by definition (3.13)]. Thus, (X,τ) is α^m-T_2-space.
(ii) By the same way of part (i) α^m-T_2-space is α^m-T_1-space and α^m-R_1-space.
Conversely, let (X,τ) be an α^m-T_1-space and α^m-R_1-space, then for each $x \neq y \in X$, there exist α^m-open sets U, V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$ implies α^m-$cl(\{x\}) \neq \alpha^m$-$cl(\{y\})$, since (X,τ) is α^m-R_1-space, then there exist disjoint α^m-open sets G, H such that $x \in G$ and $y \in H$. Thus, (X,τ) is α^m-T_2-space.

Corollary 4.14: A topological α^m-T_0-space is α^m-T_2-space if and only if for each $x \neq y \in X$ with α^m-$ker(\{x\}) \neq \alpha^m$-$ker(\{y\})$ then there exist α^m-closed sets G_1,G_2 such that α^m-$ker(\{x\}) \subseteq G_1$, α^m-$ker(\{x\}) \cap G_2 = \phi$ and α^m-$ker(\{y\}) \subseteq G_2$, α^m-$ker(\{y\}) \cap G_1 = \phi$ and $G_1 \cup G_2 = X$.

Proof: By theorem (3.19) and theorem (4.13).

Corollary 4.15: A topological α^m-T_1-space is α^m-T_2-space if and only if one of the following conditions holds:
(i) for each $x \neq y \in X$ with α^m-$cl(\{x\}) \neq \alpha^m$-$cl(\{y\})$, then there exist α^m-open sets U, V such that α^m-$cl(\alpha^m$-$ker(\{x\})) \subseteq U$ and α^m-$cl(\alpha^m$-$ker(\{y\})) \subseteq V$.
(ii) for each $x \neq y \in X$ with α^m-$ker(\{x\}) \neq \alpha^m$-$ker(\{y\})$, then there exist α^m-closed sets G_1,G_2 such that α^m-$ker(\{x\}) \subseteq G_1$, α^m-$ker(\{x\}) \cap G_2 = \phi$ and α^m-$ker(\{y\}) \subseteq G_2$, α^m-$ker(\{y\}) \cap G_1 = \phi$ and $G_1 \cup G_2 = X$.

Proof: (i) By corollary (3.20) and theorem (4.13).
(ii) By theorem (3.19) and theorem (4.13).

Theorem 4.16: A topological α^m-R_1-space is α^m-T_2-space if and only if one of the following conditions holds:
(i) for each $x \in X$, α^m-$ker(\{x\}) = \{x\}$.
(ii) for each $x \neq y \in X$, α^m-$ker(\{x\}) \neq \alpha^m$-$ker(\{y\})$ implies α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$.
(iii) for each $x \neq y \in X$, either $x \notin \alpha^m$-$ker(\{y\})$ or $y \notin \alpha^m$-$ker(\{x\})$.
(iv) for each $x \neq y \in X$ then $x \notin \alpha^m$-$ker(\{y\})$ and $y \notin \alpha^m$-$ker(\{x\})$.

Proof: (i) Let (X,τ) be an α^m-T_2-space. Then (X,τ) is α^m-T_1-space and α^m-R_1-space [by theorem (4.13)]. Hence by theorem (4.7), α^m-$ker(\{x\}) = \{x\}$ for each $x \in X$.
Conversely, let for each $x \in X$, α^m-$ker(\{x\}) = \{x\}$, then by theorem (4.7), (X,τ) is α^m-T_1-space. Also (X,τ) is α^m-R_1-space by assumption. Hence by theorem (4.13), (X,τ) is α^m-T_2-space.
(ii) Let (X,τ) be an α^m-T_2-space. Then (X,τ) is α^m-T_1-space [by remark (4.2)]. Hence by theorem (4.11), α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$ for each $x \neq y \in X$.
Conversely, assume that for each $x \neq y \in X$, α^m-$ker(\{x\}) \neq \alpha^m$-$ker(\{y\})$ implies α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$. So by theorem (4.11), (X,τ) is α^m-T_1-space, also (X,τ) is α^m-R_1-space by assumption. Hence by theorem (4.13), (X,τ) is α^m-T_2-space.
(iii) Let (X,τ) be an α^m-T_2-space. Then (X,τ) is α^m-T_0-space [by remark (4.2)]. Hence by theorem (4.3), either $x \notin \alpha^m$-$ker(\{y\})$ or $y \notin \alpha^m$-$ker(\{x\})$ for each $x \neq y \in X$.
Conversely, assume that for each $x \neq y \in X$, either $x \notin \alpha^m$-$ker(\{y\})$ or $y \notin \alpha^m$-$ker(\{x\})$ for each $x \neq y \in X$. So by theorem (4.3), (X,τ) is α^m-T_0-space, also (X,τ) is α^m-R_1-space by assumption. Thus (X,τ) is α^m-T_2-space [by theorem (4.13)].
(iv) Let (X,τ) be an α^m-T_2-space. Then (X,τ) is α^m-T_1-space and α^m-R_1-space [by theorem (4.13)]. Hence by theorem (4.10), $x \notin \alpha^m$-$ker(\{y\})$ and $y \notin \alpha^m$-$ker(\{x\})$.
Conversely, let for each $x \neq y \in X$ then $x \notin \alpha^m$-$ker(\{y\})$ and $y \notin \alpha^m$-$ker(\{x\})$. Then by theorem (4.10), (X,τ) is α^m-T_1-space. Also (X,τ) is α^m-R_1-space by assumption. Hence by theorem (4.13), (X,τ) is α^m-T_2-space.

Remark 4.17: Each α^m-separation axiom is defined as the conjunction of two weaker axioms: α^m-T_i-space $=$ α^m-R_{i-1}-space and α^m-T_{i-1}-space $=$ α^m-R_{i-1}-space and α^m-T_0-space, $i = 1,2$.

Definition 4.18: Let (X,τ) be a topological space. Then X is called:
(i) α^m-regular space ($\alpha^m r$-space, for short), if for each point x and each α^m-closed set F such that $x \in F^c$, there exist disjoint α^m-open sets U and V such that $x \in U$ and $F \subseteq V$.
(ii) α^m-normal space ($\alpha^m n$-space, for short) iff for each pair of disjoint α^m-closed sets A and B, there exist disjoint α^m-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 4.19: A topological space (X,τ) is $\alpha^m r$-space if and only if for each α^m-closed subset G of X and $x \notin G$ with $\alpha^m\text{-}ker(G) \neq \alpha^m\text{-}ker(\{x\})$ then there exist α^m-closed sets F_1,F_2 such that $\alpha^m\text{-}ker(G) \subseteq F_1$, $\alpha^m\text{-}ker(G) \cap F_2 = \phi$ and $\alpha^m\text{-}ker(\{x\}) \subseteq F_2$, $\alpha^m\text{-}ker(\{x\}) \cap F_1 = \phi$ and $F_1 \cup F_2 = X$.

Proof: Let (X,τ) be an $\alpha^m r$-space and let G be an α^m-closed set, $x \notin G$, then there exist disjoint α^m-open sets U,V such that $G \subseteq U$, $x \in V$ and $U \cap V = \phi$, then U^c and V^c are α^m-closed sets such that $U^c \cup V^c = X$. Put $F_2 = U^c$ and $F_1 = V^c$, so we get $\alpha^m\text{-}ker(G) \subseteq U \subseteq F_1$, $\alpha^m\text{-}ker(G) \cap F_2 = \phi$ and $\alpha^m\text{-}ker(\{x\}) \subseteq V \subseteq F_2$, $\alpha^m\text{-}ker(\{x\}) \cap F_1 = \phi$ and $F_1 \cup F_2 = X$.
Conversely, let for each α^m-closed subset G of X and $x \notin G$ with $\alpha^m\text{-}ker(G) \neq \alpha^m\text{-}ker(\{x\})$, then there exist α^m-closed sets F_1,F_2 such that $\alpha^m\text{-}ker(G) \subseteq F_1$, $\alpha^m\text{-}ker(G) \cap F_2 = \phi$ and $\alpha^m\text{-}ker(\{x\}) \subseteq F_2$, $\alpha^m\text{-}ker(\{x\}) \cap F_1 = \phi$ and $F_1 \cup F_2 = X$. Then F_1^c and F_2^c are α^m-open sets such that $F_1^c \cap F_2^c = \phi$ and $\alpha^m\text{-}ker(G) \cap F_1^c = \phi$, $\alpha^m\text{-}ker(\{x\}) \cap F_2^c = \phi$. So that $G \subseteq F_2^c$ and $x_\lambda \in F_1^c$. Thus, (X,τ) is $\alpha^m r$-space.

Lemma 4.20: Let (X,τ) be an $\alpha^m r$-space and F be an α^m-closed set. Then $\alpha^m\text{-}ker(F) = F = \alpha^m\text{-}cl(F)$.

Proof: Let (X,τ) be an $\alpha^m r$-space and F be an α^m-closed set. Then for each $x \notin F$, there exist disjoint α^m-open sets U,V such that $F \subseteq U$ and $x \in V$. Since $\alpha^m\text{-}ker(F) \subseteq U$, implies $\alpha^m\text{-}ker(F) \cap V = \phi$, thus $x \notin \alpha^m\text{-}cl(\alpha^m\text{-}ker(F))$. We showing that if $x \notin F$ implies $x \notin \alpha^m\text{-}cl(\alpha^m\text{-}ker(F))$, therefore $\alpha^m\text{-}cl(\alpha^m\text{-}ker(F)) \subseteq F = \alpha^m\text{-}cl(F)$. As $\alpha^m\text{-}cl(F) = F \subseteq \alpha^m\text{-}ker(F)$ [by definition (3.1)]. Thus, $\alpha^m\text{-}ker(F) = F = \alpha^m\text{-}cl(F)$.

Theorem 4.21: A topological space (X,τ) is $\alpha^m n$-space if and only if for each disjoint α^m-closed sets G,H with $\alpha^m\text{-}ker(G) \neq \alpha^m\text{-}ker(H)$ then there exist α^m-closed sets F_1,F_2 such that $\alpha^m\text{-}ker(G) \subseteq F_1$, $\alpha^m\text{-}ker(G) \cap F_2 = \phi$ and $\alpha^m\text{-}ker(H) \subseteq F_2$, $\alpha^m\text{-}ker(H) \cap F_1 = \phi$ and $F_1 \cup F_2 = X$.

Proof: Let (X,τ) be an $\alpha^m n$-space and let for each disjoint α^m-closed sets G,H with $\alpha^m\text{-}ker(G) \neq \alpha^m\text{-}ker(H)$ then there exist disjoint α^m-open sets U,V such that $G \subseteq U$ and $H \subseteq V$ and $U \cap V = \phi$, then U^c and V^c are α^m-closed sets such that $U^c \cup V^c = X$ and $\alpha^m\text{-}ker(G) \cap U^c = \phi$, $\alpha^m\text{-}ker(H) \cap V^c = \phi$. Put $U^c = F_2$ and $V^c = F_1$. Thus, $\alpha^m\text{-}ker(G) \subseteq F_1$, $\alpha^m\text{-}ker(G) \cap F_2 = \phi$ and $\alpha^m\text{-}ker(H) \subseteq F_2$, $\alpha^m\text{-}ker(H) \cap F_1 = \phi$.
Conversely, let for each disjoint α^m-closed sets G,H with $\alpha^m\text{-}ker(G) \neq \alpha^m\text{-}ker(H)$, there exist α^m-closed sets F_1,F_2 such that $\alpha^m\text{-}ker(G) \subseteq F_1$, $\alpha^m\text{-}ker(G) \cap F_2 = \phi$ and $\alpha^m\text{-}ker(H) \subseteq F_2$, $\alpha^m\text{-}ker(H) \cap F_1 = \phi$ and $F_1 \cup F_2 = X$ implies F_1^c and F_2^c are α^m-open sets such that $F_1^c \cap F_2^c = \phi$. Put $F_1^c = V$ and $F_2^c = U$, thus $\alpha^m\text{-}ker(G) \subseteq U$ and $\alpha^m\text{-}ker(H) \subseteq V$, so that $G \subseteq U$ and $H \subseteq V$. Thus (X,τ) is $\alpha^m n$-space.

Remark 4.22: The relation between α^m-separation axioms can be representing as a matrix. Therefore, the element a_{ij} refers to this relation. As the following matrix representation shows:

and	α^m-T_0	α^m-T_1	α^m-T_2	α^m-R_0	α^m-R_1
α^m-T_0	α^m-T_0	α^m-T_1	α^m-T_2	α^m-T_1	α^m-T_2
α^m-T_1	α^m-T_1	α^m-T_1	α^m-T_2	α^m-T_1	α^m-T_2
α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2
α^m-R_0	α^m-T_1	α^m-T_1	α^m-T_2	α^m-R_0	α^m-R_1
α^m-R_1	α^m-T_2	α^m-T_2	α^m-T_2	α^m-R_1	α^m-R_1

Matrix Representation (4.1)
The relation between α^m-separation axioms

5. α^m-kr-spaces

Definition 5.1: A topological space (X,τ) is said to be α^m-kr-space if and only if for each subset A of X, then $\alpha^m\text{-}ker(A)$ is an α^m-open set.

Definition 5.2: A topological α^m-kr-space (X,τ) is called α^m-T_K-space if and only if for each $x \in X$, then $\alpha^m\text{-}ker_{dr}(\{x\})$ is an α^m-open set.

Example 5.3: Let $X = \{a,b\}$ and let $\tau = \{\phi, X, \{a\}\}$ be a topology on X. Then, (X,τ) is α^m-T_K-space.

Theorem 5.4: In topological α^m-kr-space (X,τ), every α^m-T_1-space is α^m-T_K-space.

Proof: Let (X,τ) be an α^m-T_1-space. Then, for each $x \in X$, α^m-$ker(\{x\}) = \{x\}$ [by theorem (4.7)]. As α^m-$ker_{dr}(\{x\}) = \alpha^m$-$ker(\{x\}) - \{x\}$, implies α^m-$ker_{dr}(\{x\}) = \phi$. Thus, (X,τ) is α^m-T_K-space.

Theorem 5.5: In topological α^m-kr-space (X,τ), every α^m-T_K-space is α^m-T_0-space.

Proof: Let (X,τ) be an α^m-T_K-space and let $x \neq y \in X$. Then, α^m-$ker_{dr}(\{x\})$ is α^m-open set, therefore, there exist two cases:
(i) $y \in \alpha^m$-$ker_{dr}(\{x\})$ is α^m-open set. Since $x \notin \alpha^m$-$ker_{dr}(\{x\})$. Thus (X,τ) is α^m-T_0-space
(ii) $y \notin \alpha^m$-$ker_{dr}(\{x\})$, implies $y \notin \alpha^m$-$ker(\{x\})$. But α^m-$ker(\{x\})$ is α^m-open set. Thus, (X,τ) is α^m-T_0-space.

Definition 5.6: A topological α^m-kr-space (X,τ) is said to be α^m-T_L-space if and only if for each $x \neq y \in X$, α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\})$ is degenerated (empty or singleton set).

Example 5.7: Let $X = \{a,b,c\}$ and let $\tau = \{\phi, X, \{a\}, \{a,b\}, \{a,c\}\}$ be a topology on X. Then, (X,τ) is α^m-T_L-space.

Theorem 5.8: In topological α^m-kr-space (X,τ), every α^m-T_1-space is α^m-T_L-space.

Proof: Let (X,τ) be an α^m-T_1-space. Then for each $x \neq y \in X$, α^m-$ker(\{x\}) = \{x\}$ and α^m-$ker(\{y\}) = \{y\}$ [by theorem (4.7)], implies α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$. Thus (X,τ) is α^m-T_L-space.

Theorem 5.9: In topological α^m-kr-space (X,τ), every α^m-T_L-space is α^m-T_0-space.

Proof: Let (X,τ) be an α^m-T_L-space. Then for each $x \neq y \in X$, α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\})$ is degenerated (empty or singleton set). Therefore there exist three cases:
(i) α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$, implies (X,τ) is α^m-T_0-space.
(ii) α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \{x\}$ or $\{y\}$, implies $y \notin \alpha^m$-$ker(\{x\})$ or $x \notin \alpha^m$-$ker(\{y\})$, implies (X,τ) is α^m-T_0-space.
(iii) α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \{z\}$, $z \neq x \neq y$, $z \in X$, implies $y \notin \alpha^m$-$ker(\{x\})$ and $x \notin \alpha^m$-$ker(\{y\})$, implies (X,τ) is α^m-T_0-space.

Definition 5.10: A topological α^m-kr-space (X,τ) is said to be α^m-T_N-space if and only if for each $x \neq y \in X$, α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\})$ is empty or $\{x\}$ or $\{y\}$.

Example 5.11: Let $X = \{a,b,c\}$ and let $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ be a topology on X. Then, (X,τ) is α^m- T_N-space.

Example 5.12: Let $X = \mathbb{R}$ (the set of all real number) and let $\tau = \{\phi, \mathbb{R}, [a,\infty), a \in \mathbb{R}\}$ be a topology on X. Then, (X,τ) is α^m-T_0-space but not α^m-T_K, α^m-T_L or α^m-T_N spaces.

Example 5.13: Let $X = \mathbb{N}$ (the set of all natural number) and let $\tau = \{\phi, \mathbb{N}, \{n, n+1, n+2, \ldots\}, \{n+1, n+2\}, \ldots\}$ be a topology on X. Then, (X,τ) is α^m-T_K-space but not α^m-T_L or α^m-T_N spaces.

Example 5.14: Let $X = \{a,b,c\}$ and let $\tau = \{\phi, X, \{a\}, \{a,b\}, \{a,c\}\}$ be a topology on X. Then, (X,τ) is α^m-T_L-space but not α^m-T_N-space.

Theorem 5.15: In topological α^m-kr-space (X,τ), every α^m-T_1-space is α^m-T_N-space.

Proof: Let (X,τ) be an α^m-T_1-space. Then for each $x \neq y \in X$, α^m-$ker(\{x\}) = \{x\}$ and α^m-$ker(\{y\}) = \{y\}$ [by theorem (4.7)], implies α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$. Thus (X,τ) is a α^m-T_N-space.

Theorem 5.16: In topological α^m-kr-space (X,τ), every α^m-T_N-space is α^m-T_0-space.

Proof: Let (X,τ) be an α^m-T_N-space. Then for each $x \neq y \in X$, α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\})$ is degenerated (empty or singleton set). Therefore there exist two cases:
(i) α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$, implies (X,τ) is α^m-T_0-space
(ii) α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \{x\}$ or $\{y\}$, implies $y \notin \alpha^m$-$ker(\{x\})$ or $x \notin \alpha^m$-$ker(\{y\})$, implies (X,τ) is α^m-T_0-space.

Theorem 5.17: A topological α^m-kr-space (X,τ) is α^m-T_2-space iff for each $x \neq y \in X$, then α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$.

Proof: Let (X,τ) be an α^m-T_2-space. Then for each $x \neq y \in X$ there exist disjoint α^m-open sets U, V such that $x \in U$, and $y \in V$. Hence α^m-$ker(\{x\}) \subseteq U$ and α^m-$ker(\{y\}) \subseteq V$. Thus α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$.
Conversely, let for each $x \neq y \in X$, α^m-$ker(\{x\}) \cap \alpha^m$-$ker(\{y\}) = \phi$. Since (X,τ) be a topological α^m-kr-space, this means α^m-kernel is an α^m-open set. Thus (X,τ) is α^m-T_2-space.

Theorem 5.18: A topological α^m-kr-space (X,τ) is $\alpha^m r$-space iff for each G α^m-closed set and $x \notin G$, then α^m-$ker(G) \cap \alpha^m$-$ker(\{x\}) = \phi$.

Proof: By the same way of proof of theorem (5.17).

Theorem 5.19: A topological α^m-kr-space (X,τ) is $\alpha^m n$-space iff for each disjoint α^m-closed sets G, H, then α^m-$ker(G) \cap \alpha^m$-$ker(H) = \phi$.

Proof: By the same way of proof of theorem (5.17).

Theorem 5.20: A topological α^m-kr-space (X,τ) is α^m-T_1-space iff it is α^m-R_0-space and α^m-T_K-space.

Proof: By theorem (5.5) and remark (4.17).

Theorem 5.21: A topological α^m-kr-space (X,τ) is α^m-T_1-space iff it is α^m-R_0-space and α^m-T_L-space.

Proof: By theorem (5.9) and remark (4.17).

Theorem 5.22: A topological α^m-kr-space (X,τ) is α^m-T_1-space if and only if it is α^m-R_0-space and α^m-T_N-space.

Proof: By theorem (5.14) and remark (4.17).

Theorem 5.23: A topological α^m-kr-space (X,τ) is α^m-T_i-space if and only if it is α^m-R_{i-1}-space and α^m-T_K-space, $i = 1,2$.

Proof: By theorem (5.5) and remark (4.17).

Theorem 5.24: A topological α^m-kr-space (X,τ) is α^m-T_i-space if and only if it is α^m-R_{i-1}-space and α^m-T_L-space, $i = 1,2$.

Proof: By theorem (5.9) and remark (4.17).

Theorem 5.25: A topological α^m-kr-space (X,τ) is α^m-T_i-space if and only if it is α^m-R_{i-1}-space and α^m-T_N-space, $i = 1,2$.

Proof: By theorem (5.14) and remark (4.17).

Remark 5.26: The relation between α^m-separation axioms can be representing as a matrix. Therefore, the element a_{ij} refers to this relation. As the following matrix representation shows:

and	α^m-T_0	α^m-T_1	α^m-T_2	α^m-R_0	α^m-R_1	α^m-T_K	α^m-T_L	α^m-T_N
α^m-T_0	α^m-T_0	α^m-T_1	α^m-T_2	α^m-T_1	α^m-T_2	α^m-T_K	α^m-T_L	α^m-T_N
α^m-T_1	α^m-T_1	α^m-T_1	α^m-T_2	α^m-T_1	α^m-T_2	α^m-T_1	α^m-T_1	α^m-T_1
α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2	α^m-T_2
α^m-R_0	α^m-T_1	α^m-T_1	α^m-T_2	α^m-R_0	α^m-R_1	α^m-T_1	α^m-T_1	α^m-T_1
α^m-R_1	α^m-T_2	α^m-T_2	α^m-T_2	α^m-R_1	α^m-R_1	α^m-T_2	α^m-T_2	α^m-T_2
α^m-T_K	α^m-T_K	α^m-T_1	α^m-T_2	α^m-T_1	α^m-T_2	α^m-T_K	α^m-T_L	α^m-T_0
α^m-T_L	α^m-T_L	α^m-T_1	α^m-T_2	α^m-T_1	α^m-T_2	α^m-T_L	α^m-T_L	α^m-T_0
α^m-T_N	α^m-T_N	α^m-T_1	α^m-T_2	α^m-T_1	α^m-T_2	α^m-T_0	α^m-T_0	α^m-T_N

Matrix Representation (5.1)
The relation between α^m-separation axioms in topological α^m-kr-spaces

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