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Fuzzy α^m-Separation Axioms

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Abstract: **This paper will introduce a new class of fuzzy closed set (briefly F-CS) called fuzzy α^m-closed set as well as, introduce the fuzzy α^m-kernel set of the fuzzy topological space. The investigation will address and discuss some of the properties of the fuzzy separation axioms such as fuzzy α^m-R_i-space and fuzzy α^m-T_j-space (note that, the indexes i and j are natural numbers of the spaces R and T are from 0 to 3 and from 0 to 4 respectively).**

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Keywords: **Fα^m-closed set, Fα^m-R_i-space, $i = 0, 1, 2, 3$ and Fα^m-T_j-space, $j = 0, 1, 2, 3, 4$.**

1. INTRODUCTION

In 1965 Zadeh studied the fuzzy sets (briefly F-sets) (see [6]) which plays such a role in the field of fuzzy topological spaces (or simply fts). The fuzzy topological spaces investigated by Chang in 1968 (see [3]). A. S. Bin Shahna [1] defined fuzzy α-closed sets. In 1997, fuzzy generalized closed set (briefly Fg-CS) was introduced by G. Balasubramania and P. Sundaram [5]. In 2014, M. Mathew and R. Parimelazhagan [7] defined α^m-closed sets of topological spaces. The aim of this paper is to introduce a concept of Fα^m-closed sets and study their basic properties in fts. Furthermore, the investigation will include some of the properties of the fuzzy separation axioms such fuzzy α^m-R_i-space and fuzzy α^m-T_j-space (here the indexes i and j are natural numbers of the spaces R and T are from 0 to 3 and from 0 to 4 respectively).

2. PRELIMINARIES

Throughout this paper, (X,τ) or simply X always mean a fts. A fuzzy point [4] with support $x \in X$ and value λ $(0 < \lambda \leq 1)$ at $x \in X$ will be denoted by x_λ, and for fuzzy set $\mathcal{A}$, $x_\lambda \in \mathcal{A}$ iff $\lambda \leq \mathcal{A}(x)$. Two fuzzy points x_λ and y_σ are said to be distinct iff their supports are distinct. That is, by 0_X and 1_X we mean the constant fuzzy sets taking the values 0 and 1 on X, respectively [2]. For a fuzzy set $\mathcal{A}$ in a fts (X,τ), $cl(\mathcal{A})$, $int(\mathcal{A})$ and $\mathcal{A}^c = 1_X - \mathcal{A}$ represents the fuzzy closure of $\mathcal{A}$, the fuzzy interior of $\mathcal{A}$ and the fuzzy complement of $\mathcal{A}$ respectively.

Definition 2.1:[12] A fuzzy point in a set X with support x and membership value 1 is called crisp point, denoted by x_1. For any fuzzy set $\mathcal{A}$ in X, we have $x_1 \in \mathcal{A}$ iff $\mathcal{A}(x) = 1$.

Definition 2.2:[8] A fuzzy point $x_\lambda \in \mathcal{A}$ is called quasi-coincident (briefly q-coincident) with the fuzzy set $\mathcal{A}$ is denoted by $x_\lambda q\mathcal{A}$ iff $\lambda + \mathcal{A}(x) > 1$. A fuzzy set $\mathcal{A}$ in a fts (X,τ) is called q-coincident with a fuzzy set $\mathcal{B}$ which is denoted by $\mathcal{A}q\mathcal{B}$ iff there exists $x \in X$ such that $\mathcal{A}(x) + \mathcal{B}(x) > 1$. If the fuzzy sets $\mathcal{A}$ and $\mathcal{B}$ in a fts (X,τ) are not q-coincident then we write $\mathcal{A}\bar{q}\mathcal{B}$. Note that $\mathcal{A} \leq \mathcal{B} \Leftrightarrow \mathcal{A}\bar{q}(1_X - \mathcal{B})$.

Definition 2.3:[8] A fuzzy set $\mathcal{A}$ in a fts (X,τ) is called q-neighbourhood (briefly q-nhd) of a fuzzy point x_λ (resp. fuzzy set $\mathcal{B}$) if there is a F-OS $\mathcal{M}$ in a fts (X,τ) such that $x_\lambda q\mathcal{M} \leq \mathcal{A}$ (resp. $\mathcal{B}q\mathcal{M} \leq \mathcal{A}$).

Definition 2.4:[1] A fuzzy set $\mathcal{A}$ of a fts (X,τ) is called a fuzzy α-open set (briefly Fα-OS) if $\mathcal{A} \leq int(cl(int(\mathcal{A})))$ and a fuzzy α-closed set (briefly Fα-CS) if $cl(int(cl(\mathcal{A}))) \leq \mathcal{A}$. The fuzzy α-closure of a fuzzy set $\mathcal{A}$ of fts (X,τ) is the intersection of all Fα-CS that contain $\mathcal{A}$ and is denoted by $\alpha cl(\mathcal{A})$.

Definition 2.5:[5] A fuzzy set $\mathcal{A}$ of a fts (X,τ) is called a fuzzy g-closed set (briefly Fg-CS) if $cl(\mathcal{A}) \leq \mathcal{U}$ whenever $\mathcal{A} \leq \mathcal{U}$ and $\mathcal{U}$ is a F-OS in X.

Definition 2.6:[10] A fuzzy set $\mathcal{A}$ of a fts (X,τ) is called a fuzzy αg-closed set (briefly Fαg-CS) if $\alpha cl(\mathcal{A}) \leq \mathcal{U}$ whenever $\mathcal{A} \leq \mathcal{U}$ and $\mathcal{U}$ is a Fα-OS in X.

Definition 2.7:[9] A fuzzy set $\mathcal{A}$ of a fts (X,τ) is called a fuzzy gα-closed set (briefly Fgα-CS) if $\alpha cl(\mathcal{A}) \leq \mathcal{U}$ whenever $\mathcal{A} \leq \mathcal{U}$ and $\mathcal{U}$ is a F-OS in X.

Remark 2.8:[5,11] In a fts (X,τ), then the following statements are true:
(i) Every F-CS is a Fg-CS.
(ii) Every F-CS is a Fα-CS.

Remark 2.9:[9,10] In a fts (X,τ), then the following statements are true:
(i) Every Fg-CS is a Fgα-CS.
(ii) Every Fα-CS is a Fαg-CS.
(iii) Every Fαg-CS is a Fgα-CS.

3. FUZZY α^m-CLOSED SETS

Definition 3.1: A fuzzy set $\mathcal{A}$ of a fts (X,τ) is called a fuzzy α^m-closed set (briefly Fα^m-CS) if $int(cl(\mathcal{A})) \leq \mathcal{U}$ whenever $\mathcal{A} \leq \mathcal{U}$ and $\mathcal{U}$ is a Fα-OS. The complement of a fuzzy α^m-closed set in X is fuzzy α^m-open set (briefly Fα^m-OS) in X, the family of all Fα^m-OS (resp. Fα^m-CS) of a fts (X,τ) is denoted by Fα^m-$O(X)$ (resp. Fα^m-$C(X)$).

Example 3.2: Let $X = \{x,y\}$ and the fuzzy set $\mathcal{A}$ in X defined as follows: $\mathcal{A}(x) = 0.5, \mathcal{A}(y) = 0.5$.
Let $\tau = \{0_X, \mathcal{A}, 1_X\}$ be a fts. Then the fuzzy sets $0_X, \mathcal{A}$ and 1_X are Fα^m-OS and Fα^m-CS at the same time in X.

Remark 3.3: In a fts (X,τ), then the following statements are true:
(i) Every F-CS is a Fα^m-CS.
(ii) Every Fα^m-CS is a Fα-CS.
(iii) Every Fα^m-CS is a Fαg-CS.
(iv) Every Fα^m-CS is a Fgα-CS.

Proof: (i) This follows directly from the definition (3.1).
(ii) Let $\mathcal{A}$ be a Fα^m-CS in X and let $\mathcal{U}$ be a F-OS such that $\mathcal{A} \leq \mathcal{U}$. Since every F-OS is a Fα-OS and $\mathcal{A}$ is a Fα^m-CS, $int(cl(\mathcal{A})) \leq (int(cl(\mathcal{A}))) \vee (cl(int(\mathcal{A}))) \leq \mathcal{U}$. Therefore, $\mathcal{A}$ is a Fα-CS in X.
(iii) From the part (ii) and remark (2.9) (ii).
(iv) From the part (iii) and remark (2.9) (iii).

Theorem 3.4: A fuzzy set $\mathcal{A}$ is Fα^m-CS iff $int(cl(\mathcal{A})) - \mathcal{A}$ contains no non-empty Fα^m-CS.

Proof: Necessity. Suppose that $\mathcal{F}$ is a non-empty Fα^m-closed subset of $int(cl(\mathcal{A}))$ such that $\mathcal{F} < int(cl(\mathcal{A})) - \mathcal{A}$. Then $\mathcal{F} \leq int(cl(\mathcal{A})) - \mathcal{A}$. Then $\mathcal{F} \leq int(cl(\mathcal{A})) \wedge \mathcal{A}^c$. Therefore $\mathcal{F} \leq int(cl(\mathcal{A}))$ and $\mathcal{F} \leq \mathcal{A}^c$. Since $\mathcal{F}^c$ is a Fα^m-OS and $\mathcal{A}$ is a Fα^m-CS, $int(cl(\mathcal{A})) \leq \mathcal{F}^c$. Thus $\mathcal{F} \leq (int(cl(\mathcal{A})))^c$. Therefore $\mathcal{F} \leq (int(cl(\mathcal{A}))) \wedge (int(cl(\mathcal{A})))^c = 0_X$. Therefore $\mathcal{F} = 0_X \Longrightarrow int(cl(\mathcal{A})) - \mathcal{A}$ contains no non-empty Fα^m-CS.

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Sufficiency. Let $\mathcal{A} \leq \mathcal{U}$ be a Fα^m-OS. Suppose that $int(cl(\mathcal{A}))$ is not contained in $\mathcal{U}$. Then $(int(cl(\mathcal{A})))^c$ is a non-empty Fα^m-CS and contained in $int(cl(\mathcal{A})) - \mathcal{A}$ which is a contradiction. Therefore, $int(cl(\mathcal{A})) \leq \mathcal{U}$ and hence $\mathcal{A}$ is a Fα^m-CS.

Theorem 3.5: Let $\mathcal{B} \leq Y \leq X$, if $\mathcal{B}$ is a Fα^m-CS relative to Y and Y is a F-OS then $\mathcal{B}$ is a Fα^m-CS in a fts (X,τ).

Proof: Let $\mathcal{U}$ be a Fα-OS in a fts (X,τ) such that $\mathcal{B} \leq \mathcal{U}$. Given that $\mathcal{B} \leq Y \leq X$. Therefore $\mathcal{B} \leq Y$ and $\mathcal{B} \leq \mathcal{U}$. This implies $\mathcal{B} \leq Y \wedge \mathcal{U}$. Since $\mathcal{B}$ is a Fα^m-CS relative to Y, then $int(cl(\mathcal{B})) \leq \mathcal{U}. Y \wedge int(cl(\mathcal{B})) \leq Y \wedge \mathcal{U}$ implies that $Y \wedge (int(cl(\mathcal{B}))) \leq \mathcal{U}$. Thus $[Y \wedge int(cl(\mathcal{B}))] \vee [int(cl(\mathcal{B}))]^c \leq \mathcal{U} \vee [int(cl(\mathcal{B}))]^c$. This implies that $(Y \vee (int(cl(\mathcal{B})))^c) \wedge (int(cl(\mathcal{B}))) \vee (int(cl(\mathcal{B})))^c \leq \mathcal{U} \vee (int(cl(\mathcal{B})))^c$. Therefore $(Y < (int(cl(\mathcal{B})))^c) \leq \mathcal{U} \vee (int(cl(\mathcal{B})))^c$. Since Y is a F-OS in X. $int(cl(Y)) \leq \mathcal{U} \vee (int(cl(\mathcal{B})))^c$. Also $\mathcal{B} \leq Y$ implies that $int(cl(\mathcal{B})) \leq int(cl(Y))$. Thus $nt(cl(\mathcal{B})) \leq int(cl(Y)) \leq \mathcal{U} \vee (int(cl(\mathcal{B})))^c$. Therefore $int(cl(\mathcal{B})) \leq \mathcal{U}$. Since $int(cl(\mathcal{B}))$ is not contained in $(int(cl(\mathcal{B})))^c$, $\mathcal{B}$ is a Fα^m-CS relative to X.

Theorem 3.6: If $\mathcal{A}$ is a Fα^m-CS and $\mathcal{A} \leq \mathcal{B} \leq int(cl(\mathcal{A}))$, then $\mathcal{B}$ is a Fα^m-CS.

Proof: Let $\mathcal{A}$ be a Fα^m-CS such that $\mathcal{A} \leq \mathcal{B} \leq int(cl(\mathcal{A}))$. Let $\mathcal{U}$ be a Fα-OS in a fts (X,τ) such that $\mathcal{B} \leq \mathcal{U}$. Since $\mathcal{A}$ is a Fα^m-CS, we have $int(cl(\mathcal{A})) \leq \mathcal{U}$ whenever $\mathcal{A} \leq \mathcal{U}$. Since $\mathcal{A} \leq \mathcal{B}$ and $\mathcal{B} \leq int(cl(\mathcal{A}))$, then $int(cl(\mathcal{B})) \leq int(cl(int(cl(\mathcal{A})))) \leq int(cl(\mathcal{A})) \leq \mathcal{U}$. Therefore, $nt(cl(\mathcal{B})) \leq \mathcal{U}$. Thus, $\mathcal{B}$ is a Fα^m-CS in X.

Theorem 3.7: The intersection of a Fα^m-CS and a F-CS is a Fα^m-CS.

Proof: Let $\mathcal{A}$ be a Fα^m-CS and $\mathcal{F}$ be a F-CS. Since $\mathcal{A}$ is a Fα^m-CS, $int\big(cl(\mathcal{A})\big) \leq \mathcal{U}$ whenever $\mathcal{A} \leq \mathcal{U}$ where $\mathcal{U}$ is a Fα-OS. To show that $\mathcal{A} \wedge \mathcal{F}$ is a Fα^m-CS. It is enough to show that $int\big(cl(\mathcal{A} \wedge \mathcal{F})\big) \leq \mathcal{U}$ whenever $\mathcal{A} \wedge \mathcal{F} \leq \mathcal{U}$, where $\mathcal{U}$ is a Fα-OS. Let $\mathcal{M} = 1_X - \mathcal{F}$ then $\mathcal{A} \leq \mathcal{U} \vee \mathcal{M}$. Since $\mathcal{M}$ is a F-OS, $\mathcal{U} \vee \mathcal{M}$ is a Fα-OS and $\mathcal{A}$ is a Fα^m-CS, $int\big(cl(\mathcal{A})\big) \leq \mathcal{U} \vee \mathcal{M}$. Now, $int\big(cl(\mathcal{A} \wedge \mathcal{F})\big) \leq int\big(cl(\mathcal{A})\big) \wedge int\big(cl(\mathcal{F})\big) \leq int\big(cl(\mathcal{A})\big) \wedge \mathcal{F} \leq (\mathcal{U} \vee \mathcal{M}) \wedge \mathcal{F} \leq (\mathcal{U} \wedge \mathcal{F}) \vee (\mathcal{M} \wedge \mathcal{F}) \leq (\mathcal{U} \wedge \mathcal{F}) \vee 0_X \leq \mathcal{U}$. This implies that $\mathcal{A} \wedge \mathcal{F}$ is a Fα^m-CS.

Theorem 3.8: If $\mathcal{A}$ and $\mathcal{B}$ are two Fα^m-CS in a fts (X,τ), then $\mathcal{A} \wedge \mathcal{B}$ is a Fα^m-CS in X.

Proof: Let $\mathcal{A}$ and $\mathcal{B}$ be two Fα^m-CS in a fts (X,τ). Let $\mathcal{U}$ be a Fα-OS in X such that $\mathcal{A} \wedge \mathcal{B} \leq \mathcal{U}$.
Now, $int\big(cl(\mathcal{A} \wedge \mathcal{B})\big) \leq int\big(cl(\mathcal{A})\big) \wedge int\big(cl(\mathcal{B})\big) \leq \mathcal{U}$. Hence $\mathcal{A} \wedge \mathcal{B}$ is a Fα^m-CS.

Remark 3.9: The union of two Fα^m-CS need not be a Fα^m-CS.

Definition 3.10: The intersection of all Fα^m-CS in a fts (X,τ) containing $\mathcal{A}$ is called fuzzy α^m-closure of $\mathcal{A}$ and is denoted by α^m-$cl(\mathcal{A})$, α^m-$cl(\mathcal{A}) = \wedge \{\mathcal{B} : \mathcal{A} \leq \mathcal{B}, \mathcal{B}$ is a Fα^m-CS$\}$.

Definition 3.11: The union of all Fα^m-OS in a fts (X,τ) contained in $\mathcal{A}$ is called fuzzy α^m-interior of $\mathcal{A}$ and is denoted by α^m-$int(\mathcal{A})$, α^m-$int(\mathcal{A}) = \vee \{\mathcal{B} : \mathcal{A} \geq \mathcal{B}, \mathcal{B}$ is a Fα^m-OS$\}$.

Proposition 3.12: Let $\mathcal{A}$ be any fuzzy set in a fts (X,τ). Then the following properties hold:
(i) α^m-$int(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A}$ is a Fα^m-OS.
(ii) α^m-$cl(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A}$ is a Fα^m-CS.
(iii) α^m-$int(\mathcal{A})$ is the largest Fα^m-OS contained in $\mathcal{A}$.
(iv) α^m-$cl(\mathcal{A})$ is the smallest Fα^m-CS containing $\mathcal{A}$.

Proof: (i), (ii), (iii) and (iv) are obvious.

Proposition 3.13: Let $\mathcal{A}$ be any fuzzy set in a fts (X,τ). Then the following properties hold:
(i) $\alpha^m\text{-}int(1_X-\mathcal{A})=1_X-(\alpha^m\text{-}cl(\mathcal{A}))$,
(ii) $\alpha^m\text{-}cl(1_X-\mathcal{A})=1_X-(\alpha^m\text{-}int(\mathcal{A}))$.

Proof: (i) By definition, $\alpha^m\text{-}cl(\mathcal{A})=\wedge\{\mathcal{B}:\mathcal{A}\le\mathcal{B},\mathcal{B}$ is a Fα^m-CS$\}$

$$\begin{aligned}1_X-(\alpha^m\text{-}cl(\mathcal{A}))&=1_X-\wedge\{\mathcal{B}:\mathcal{A}\le\mathcal{B},\mathcal{B}\text{ is a F}\alpha^m\text{-CS}\}\\&=\vee\{1_X-\mathcal{B}:\mathcal{A}\le\mathcal{B},\mathcal{B}\text{ is a F}\alpha^m\text{-CS}\}\\&=\vee\{\mathcal{H}:1_X-\mathcal{A}\ge\mathcal{H},\mathcal{H}\text{ is a F}\alpha^m\text{-OS}\}\\&=\alpha^m\text{-}int(1_X-\mathcal{A})\end{aligned}$$

(ii) The proof is similar to (i).

Definition 3.14: A fuzzy set $\mathcal{A}$ in a fts (X,τ) is said to be a fuzzy α^m-neighbourhood (briefly Fα^m-nhd) of a fuzzy point x_λ if there exists a Fα^m-OS $\mathcal{B}$ such that $x_\lambda\in\mathcal{B}\le\mathcal{A}$. A F$\alpha^m$-nhd $\mathcal{A}$ is said to be a Fα^m-open-nhd (resp. Fα^m-closed-nhd) iff $\mathcal{A}$ is a Fα^m-OS (resp. Fα^m-CS). A fuzzy set $\mathcal{A}$ in a fts (X,τ) is said to be a fuzzy α^m-q-neighbourhood (briefly Fα^m-q-nhd) of a fuzzy point x_λ (resp. fuzzy set $\mathcal{B}$) if there exists a Fα^m-OS $\mathcal{M}$ in a fts (X,τ) such that $x_\lambda q\mathcal{M}\le\mathcal{A}$ (resp. $\mathcal{B}q\mathcal{M}\le\mathcal{A}$).

Theorem 3.15: A fuzzy set $\mathcal{A}$ of a fts (X,τ) is Fα^m-CS iff $\mathcal{A}\bar{q}\mathcal{K}\Rightarrow int(cl(\mathcal{A}))\bar{q}\mathcal{K}$, for every F$\alpha$-CS $\mathcal{K}$ of X.

Proof: Necessity. Let $\mathcal{K}$ be a Fα-CS and $\mathcal{A}\bar{q}\mathcal{K}$. Then $\mathcal{A}\le 1_X-\mathcal{K}$ and $1_X-\mathcal{K}$ is a Fα-OS in X which implies that $int(cl(\mathcal{A}))\le 1_X-\mathcal{K}$ as $\mathcal{A}$ is a Fα^m-CS. Hence, $int(cl(\mathcal{A}))\bar{q}\mathcal{K}$.
Sufficiency. Let $\mathcal{U}$ be a Fα-OS of a fts (X,τ) such that $\mathcal{A}\le\mathcal{U}$. Then $\mathcal{A}\bar{q}(1_X-\mathcal{U})$ and $1_X-\mathcal{U}$ is a Fα-CS in X. By hypothesis, $int(cl(\mathcal{A}))\bar{q}(1_X-\mathcal{U})$ implies $int(cl(\mathcal{A}))\le\mathcal{U}$. Hence, $\mathcal{A}$ is a Fα^m-CS in X.

Theorem 3.16: Let x_λ and $\mathcal{A}$ be a fuzzy point and a fuzzy set respectively in a fts (X,τ). Then $x_\lambda\in\alpha^m\text{-}cl(\mathcal{A})$ iff every Fα^m-q-nhd of x_λ is q-coincident with $\mathcal{A}$.

Proof: Let $x_\lambda\in\alpha^m\text{-}cl(\mathcal{A})$. Suppose there exists a Fα^m-q-nhd $\mathcal{M}$ of x_λ such that $\mathcal{M}\bar{q}\mathcal{A}$. Since $\mathcal{M}$ is a Fα^m-q-nhd of x_λ, there exists a Fα^m-OS $\mathcal{N}$ in X such that $x_\lambda q\mathcal{N}\le\mathcal{M}$ whish gives that $\mathcal{N}\bar{q}\mathcal{A}$ and hence $\mathcal{A}\le 1_X-\mathcal{N}$. Then $\alpha^m\text{-}cl(\mathcal{A})\le 1_X-\mathcal{N}$, as $1_X-\mathcal{N}$ is a Fα^m-CS. Since $x_\lambda\notin 1_X-\mathcal{N}$, we have $x_\lambda\notin\alpha^m\text{-}cl(\mathcal{A})$, a contradiction. Thus every Fα^m-q-nhd of x_λ is q-coincident with $\mathcal{A}$.
Conversely, suppose $x_\lambda\notin\alpha^m\text{-}cl(\mathcal{A})$. Then there exists a F$\alpha^m$-CS $\mathcal{B}$ such that $\mathcal{A}\le\mathcal{B}$ and $x_\lambda\notin\mathcal{B}$. Then we have $x_\lambda q(1_X-\mathcal{B})$ and $\mathcal{A}\bar{q}(1_X-\mathcal{B})$, a contradiction. Hence $x_\lambda\in g\alpha g\text{-}cl(\mathcal{A})$.

Theorem 3.17: Let $\mathcal{A}$ and $\mathcal{B}$ be two fuzzy sets in a fts (X,τ). Then the following are true:
(i) $\alpha^m\text{-}cl(0_X)=0_X$, $\alpha^m\text{-}cl(1_X)=1_X$.
(ii) $\alpha^m\text{-}cl(\mathcal{A})$ is a Fα^m-CS in X.
(iii) $\alpha^m\text{-}cl(\mathcal{A})\le\alpha^m\text{-}cl(\mathcal{B})$ when $\mathcal{A}\le\mathcal{B}$.
(iv) $\mathcal{M}q\mathcal{A}$ iff $\mathcal{M}q\alpha^m\text{-}cl(\mathcal{A})$, when $\mathcal{M}$ is a Fα^m-OS in X.
(v) $\alpha^m\text{-}cl(\mathcal{A})=\alpha^m\text{-}cl(\alpha^m\text{-}cl(\mathcal{A}))$.

Proof: (i) and (ii) are obvious.
(iii) Let $x_\lambda\notin\alpha^m\text{-}cl(\mathcal{B})$. By theorem (3.16), there is a Fα^m-q-nhd $\mathcal{N}$ of a fuzzy point x_λ such that $\mathcal{N}\bar{q}\mathcal{B}$, so there is a F$\alpha^m$-OS $\mathcal{M}$ such that $x_\lambda q\mathcal{M}\le\mathcal{N}$ and $\mathcal{M}\bar{q}\mathcal{B}$. Since $\mathcal{A}\le\mathcal{B}$, then $\mathcal{M}\bar{q}\mathcal{A}$. Hence $x_\lambda\notin\alpha^m\text{-}cl(\mathcal{A})$ by theorem (3.16). Thus $\alpha^m\text{-}cl(\mathcal{A})\le\alpha^m\text{-}cl(\mathcal{B})$.
(iv) Let $\mathcal{M}$ be a Fα^m-OS in X. Suppose that $\mathcal{M}\bar{q}\mathcal{A}$, then $\mathcal{A}\le 1_X-\mathcal{M}$. Since $1_X-\mathcal{M}$ is a Fα^m-CS and by a part (iii), $\alpha^m\text{-}cl(\mathcal{A})\le\alpha^m\text{-}cl(1_X-\mathcal{M})=1_X-\mathcal{M}$. Hence, $\mathcal{M}\bar{q}\alpha^m\text{-}cl(\mathcal{A})$.
Conversely, suppose that $\mathcal{M}\bar{q}\alpha^m\text{-}cl(\mathcal{A})$. Then $\alpha^m\text{-}cl(\mathcal{A})\le 1_X-\mathcal{M}$. Since $\mathcal{A}\le\alpha^m\text{-}cl(\mathcal{A})$, we have $\mathcal{A}\le 1_X-\mathcal{M}$. Hence $\mathcal{M}\bar{q}\mathcal{A}$.

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(v) Since $\alpha^m\text{-}cl(\mathcal{A}) \leq \alpha^m\text{-}cl(\alpha^m\text{-}cl(\mathcal{A}))$. We prove that $\alpha^m\text{-}cl(\alpha^m\text{-}cl(\mathcal{A})) \leq \alpha^m\text{-}cl(\mathcal{A})$. Suppose that $x_\lambda \notin \alpha^m\text{-}cl(\mathcal{A})$. Then by theorem (3.16), there exists a Fα^m-q-nhd $\mathcal{N}$ of a fuzzy point x_λ such that $\mathcal{N}\bar{q}\mathcal{A}$ and so there is a Fα^m-OS $\mathcal{M}$ in X such that $x_\lambda q\mathcal{M} \leq \mathcal{N}$ and $\mathcal{M}\bar{q}\mathcal{A}$. By a part (iv), $\mathcal{M}\bar{q}\alpha^m\text{-}cl(\mathcal{A})$. Then by theorem (3.16), $x_\lambda \notin \alpha^m\text{-}cl(\alpha^m\text{-}cl(\mathcal{A}))$. Thus $\alpha^m\text{-}cl(\alpha^m\text{-}cl(\mathcal{A})) \leq \alpha^m\text{-}cl(\mathcal{A})$. Hence $\alpha^m\text{-}cl(\mathcal{A}) = \alpha^m\text{-}cl(\alpha^m\text{-}cl(\mathcal{A}))$.

Theorem 3.18: Let $\mathcal{A}$ and $\mathcal{B}$ be two fuzzy sets in a fts (X,τ). Then the following are true:
(i) $\alpha^m\text{-}int(0_X) = 0_X$, $\alpha^m\text{-}int(1_X) = 1_X$.
(ii) $\alpha^m\text{-}int(\mathcal{A})$ is a Fα^m-OS in X.
(iii) $\alpha^m\text{-}int(\mathcal{A}) \leq \alpha^m\text{-}int(\mathcal{A})$ when $\mathcal{A} \leq \mathcal{B}$.
(iv) $\alpha^m\text{-}int(\mathcal{A}) = \alpha^m\text{-}int(\alpha^m\text{-}int(\mathcal{A}))$.

Proof: Obvious.

Remark 3.19: The following are the implications of a Fα^m-CS and the reverse is not true.

$$\begin{tikzcd}
\text{Fg-CS} \arrow[r] & \text{Fg}\alpha\text{-CS} & \text{F}\alpha\text{g-CS} \arrow[l] \\
\text{F-CS} \arrow[u] \arrow[r] & \text{F}\alpha^m\text{-CS} \arrow[u] \arrow[r] & \text{F}\alpha\text{-CS} \arrow[u]
\end{tikzcd}$$

4. FUZZY α^m-KERNEL AND FUZZY α^m-R_i-SPACES, $i = 0, 1, 2, 3$

Definition 4.1: The intersection of all Fα^m-open subset of X containing $\mathcal{M}$ is called the fuzzy α^m-kernel of $\mathcal{M}$ (briefly $\alpha^m\text{-}ker(\mathcal{M})$), this means $\alpha^m\text{-}ker(\mathcal{M}) = \wedge\{\mathcal{U} \in \text{F}\alpha^m\text{-}O(X): \mathcal{M} \leq \mathcal{U}\}$.

Definition 4.2: In a fts (X,τ), a fuzzy set $\mathcal{M}$ is said to be weakly ultra fuzzy α^m-separated from $\mathcal{N}$ if there exists a Fα^m-OS $\mathcal{U}$ such that $\mathcal{U} \wedge \mathcal{N} = 0_X$ or $\mathcal{M} \wedge \alpha^m\text{-}cl(\mathcal{N}) = 0_X$.

By definition (4.2), we have the following: For every two distinct fuzzy points x_λ and y_σ of X,
(i) $\alpha^m\text{-}cl(\{x_\lambda\}) = \{y_\sigma : \{y_\sigma\}$ is not weakly ultra fuzzy α^m-separated from $\{x_\lambda\}\}$.
(ii) $\alpha^m\text{-}ker(\{x_\lambda\}) = \{y_\sigma : \{x_\lambda\}$ is not weakly ultra fuzzy α^m-separated from $\{y_\sigma\}\}$.

Corollary 4.3: Let (X,τ) be a fts, then $y_\sigma \in \alpha^m\text{-}ker(\{x_\lambda\})$ iff $x_\lambda \in \alpha^m\text{-}cl(\{y_\sigma\})$ for each $x \neq y \in X$.

Proof: Suppose that $y_\sigma \notin \alpha^m\text{-}ker(\{x_\lambda\})$. Then there exists a F$\alpha^m$-OS $\mathcal{U}$ containing x_λ such that $y_\sigma \notin \mathcal{U}$. Therefore, we have $x_\lambda \notin \alpha^m\text{-}cl(\{y_\sigma\})$. The converse part can be proved in a similar way.

Definition 4.4: A fts (X,τ) is called fuzzy α^m-R_0-space (Fα^m-R_0-space, for short) if for each Fα^m-OS $\mathcal{U}$ and $x_\lambda \in \mathcal{U}$, then $\alpha^m\text{-}cl(\{x_\lambda\}) \leq \mathcal{U}$.

Definition 4.5: A fts (X,τ) is called fuzzy α^m-R_1-space (Fα^m-R_1-space, for short) if for each two distinct fuzzy points x_λ and y_σ of X with $\alpha^m\text{-}cl(\{x_\lambda\}) \neq \alpha^m\text{-}cl(\{y_\sigma\})$, there exist disjoint F$\alpha^m$-OS $\mathcal{U},\mathcal{V}$ such that $\alpha^m\text{-}cl(\{x_\lambda\}) \leq \mathcal{U}$ and $\alpha^m\text{-}cl(\{y_\sigma\}) \leq \mathcal{V}$.

Theorem 4.6: Let (X,τ) be a fts. Then (X,τ) is Fα^m-R_0-space iff $\alpha^m\text{-}cl(\{x_\lambda\}) = \alpha^m\text{-}ker(\{x_\lambda\})$, for each $x \in X$.

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Proof: Let (X,τ) be a Fα^m-R_0-space. If α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$ker(\{x_\lambda\})$, for each $x \in X$, then there exist another fuzzy point $y \neq x$ such that $y_\sigma \in \alpha^m$-$cl(\{x_\lambda\})$ and $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ this means there exist an $\mathcal{U}_{x_\lambda}$ Fα^m-OS, $y_\sigma \notin \mathcal{U}_{x_\lambda}$ implies α^m-$cl(\{x_\lambda\}) \nleq \mathcal{U}_{x_\lambda}$ this contradiction. Thus α^m-$cl(\{x_\lambda\}) = \alpha^m$-$ker(\{x_\lambda\})$.
Conversely, let α^m-$cl(\{x_\lambda\}) = \alpha^m$-$ker(\{x_\lambda\})$, for each F$\alpha^m$-OS $\mathcal{U}, x_\lambda \in \mathcal{U}$, then α^m-$ker(\{x_\lambda\}) = \alpha^m$-$cl(\{x_\lambda\}) \leq \mathcal{U}$ [by definition (4.1)]. Hence by definition (4.4), (X,τ) is a Fα^m-R_0-space.

Theorem 4.7: A fts (X,τ) is an Fα^m-R_0-space iff for each $\mathcal{A}$ Fα^m-CS and $x_\lambda \in \mathcal{A}$, then α^m-$ker(\{x_\lambda\}) \leq \mathcal{A}$.

Proof: Let for each $\mathcal{A}$ Fα^m-CS and $x_\lambda \in \mathcal{A}$, then α^m-$ker(\{x_\lambda\}) \leq \mathcal{A}$ and let $\mathcal{U}$ be a Fα^m-OS, $x_\lambda \in \mathcal{U}$ then for each $y_\sigma \notin \mathcal{U}$ implies $y_\sigma \in \mathcal{U}^c$ is a Fα^m-CS implies α^m-$ker(\{y_\sigma\}) \leq \mathcal{U}^c$ [by assumption]. Therefore $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ implies $y_\sigma \notin \alpha^m$-$cl(\{x_\lambda\})$ [by corollary (4.3)]. So α^m-$cl(\{x_\lambda\}) \leq \mathcal{U}$. Thus (X,τ) is an Fα^m-R_0-space.
Conversely, let (X,τ) be a Fα^m-R_0-space and $\mathcal{A}$ be a Fα^m-CS and $x_\lambda \in \mathcal{A}$. Then for each $y_\sigma \notin \mathcal{A}$ implies $y_\sigma \in \mathcal{A}^c$ is a Fα^m-OS, then α^m-$cl(\{y_\sigma\}) \leq \mathcal{A}^c$[since (X,τ) is a Fα^m-R_0-space], so α^m-$ker(\{x_\lambda\}) = \alpha^m$-$cl(\{x_\lambda\})$. Thus α^m-$ker(\{x_\lambda\}) \leq \mathcal{A}$.

Corollary 4.8: A fts (X,τ) is Fα^m-R_0-space iff for each $\mathcal{U}$ Fα^m-OS and $x_\lambda \in \mathcal{U}$, then α^m-$cl(\alpha^m$-$ker(\{x_\lambda\})) \leq \mathcal{U}$.

Proof: Clearly.

Theorem 4.9: Every Fα^m-R_1-space is a Fα^m-R_0-space.

Proof: Let (X,τ) be a Fα^m-R_1-space and let $\mathcal{U}$ be a Fα^m-OS, $x_\lambda \in \mathcal{U}$, then for each $y_\sigma \notin \mathcal{U}$ implies $y_\sigma \in \mathcal{U}^c$ is a Fα^m-CS and α^m-$cl(\{y_\sigma\}) \leq \mathcal{U}^c$ implies α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$cl(\{y_\sigma\})$. Hence by definition (4.5), α^m-$cl(\{x_\lambda\}) \leq \mathcal{U}$. Thus (X,τ) is a Fα^m-R_0-space.

Theorem 4.10: A fts (X,τ) is Fα^m-R_1-space iff for each $x \neq y \in X$ with α^m-$ker(\{x_\lambda\}) \neq \alpha^m$-$ker(\{y_\sigma\})$, then there exist F$\alpha^m$-CS $\mathcal{A}_1$, $\mathcal{A}_2$ such that α^m-$ker(\{x_\lambda\}) \leq \mathcal{A}_1$, α^m-$ker(\{x_\lambda\}) \wedge \mathcal{A}_2 = 0_X$ and α^m-$ker(\{y_\sigma\}) \leq \mathcal{A}_2$, α^m-$ker(\{y_\sigma\}) \wedge \mathcal{A}_1 = 0_X$ and $\mathcal{A}_1 \vee \mathcal{A}_2 = 1_X$.

Proof: Let (X,τ) be a Fα^m-R_1-space. Then for each $x \neq y \in X$ with α^m-$ker(\{x_\lambda\}) \neq \alpha^m$-$ker(\{y_\sigma\})$. Since every F$\alpha^m$-$R_1$-space is a F$\alpha^m$-$R_0$-space [by theorem (4.9)], and by theorem (4.6), α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$cl(\{y_\sigma\})$, then there exist F$\alpha^m$-OS $\mathcal{U}_1$,$\mathcal{U}_2$ such that α^m-$cl(\{x_\lambda\}) \leq \mathcal{U}_1$ and α^m-$cl(\{y_\sigma\}) \leq \mathcal{U}_2$ and $\mathcal{U}_1 \wedge \mathcal{U}_2 = 0_X$ [since (X,τ) is a Fα^m-R_1-space], then $\mathcal{U}_1^c$ and $\mathcal{U}_2^c$ are Fα^m-CS such that $\mathcal{U}_1^c \vee \mathcal{U}_2^c = 1_X$. Put $\mathcal{A}_1 = \mathcal{U}_1^c$ and $\mathcal{A}_2 = \mathcal{U}_2^c$. Thus $x_\lambda \in \mathcal{U}_1 \leq \mathcal{A}_2$ and $y_\sigma \in \mathcal{U}_2 \leq \mathcal{A}_1$ so that α^m-$ker(\{x_\lambda\}) \leq \mathcal{U}_1 \leq \mathcal{A}_2$ and α^m-$ker(\{y_\sigma\}) \leq \mathcal{U}_2 \leq \mathcal{A}_1$.
Conversely, let for each $x \neq y \in X$ with α^m-$ker(\{x_\lambda\}) \neq \alpha^m$-$ker(\{y_\sigma\})$, there exist F$\alpha^m$-CS $\mathcal{A}_1$, $\mathcal{A}_2$ such that α^m-$ker(\{x_\lambda\}) \leq \mathcal{A}_1, \alpha^m$-$ker(\{x_\lambda\}) \wedge \mathcal{A}_2 = 0_X$ and α^m-$ker(\{y_\sigma\}) \leq \mathcal{A}_2$, α^m-$ker(\{y_\sigma\}) \wedge \mathcal{A}_1 = 0_X$ and $\mathcal{A}_1 \vee \mathcal{A}_2 = 1_X$, then $\mathcal{A}_1^c$ and $\mathcal{A}_2^c$ are Fα^m-OS such that $\mathcal{A}_1^c \wedge \mathcal{A}_2^c = 0_X$. Put $\mathcal{A}_1^c = \mathcal{U}_2$ and $\mathcal{A}_2^c = \mathcal{U}_1$. Thus, α^m-$ker(\{x_\lambda\}) \leq \mathcal{U}_1$ and α^m-$ker(\{y_\sigma\}) \leq \mathcal{U}_2$ and $\mathcal{U}_1 \wedge \mathcal{U}_2 = 0_X$, so that $x_\lambda \in \mathcal{U}_1$ and $y_\sigma \in \mathcal{U}_2$ implies $x_\lambda \notin \alpha^m$-$cl(\{y_\sigma\})$ and $y_\sigma \notin \alpha^m$-$cl(\{x_\lambda\})$, then α^m-$cl(\{x_\lambda\}) \leq \mathcal{U}_1$ and α^m-$cl(\{y_\sigma\}) \leq \mathcal{U}_2$. Thus, (X,τ) is a Fα^m-R_1-space.

Corollary 4.11: A fts (X,τ) is Fα^m-R_1-space iff for each $x \neq y \in X$ with α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$cl(\{y_\sigma\})$ there exist disjoint Fα^m-OS $\mathcal{U}, \mathcal{V}$ such that α^m-$cl(\alpha^m$-$ker(\{x_\lambda\})) \leq \mathcal{U}$ and α^m-$cl(\alpha^m$-$ker(\{y_\sigma\})) \leq \mathcal{V}$.

Proof: Let (X,τ) be a Fα^m-R_1-space and let $x \neq y \in X$ with α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$cl(\{y_\sigma\})$, then there exist disjoint Fα^m-OS $\mathcal{U}, \mathcal{V}$ such that α^m-$cl(\{x_\lambda\}) \leq \mathcal{U}$ and α^m-$cl(\{y_\sigma\}) \leq \mathcal{V}$. Also (X,τ) is Fα^m-R_0-space [by theorem (4.9)] implies for each $x \in X$, then α^m-$cl(\{x_\lambda\}) = \alpha^m$-$ker(\{x_\lambda\})$ [by theorem (4.6)], but α^m-$cl(\{x_\lambda\}) = \alpha^m$-$cl(\alpha^m$-$cl(\{x_\lambda\})) = \alpha^m$-$cl(\alpha^m$-$ker(\{x_\lambda\}))$. Thus α^m-$cl(\alpha^m$-$ker(\{x_\lambda\})) \leq \mathcal{U}$ and α^m-$cl(\alpha^m$-$ker(\{y_\sigma\})) \leq \mathcal{V}$.
Conversely, let for each $x \neq y \in X$ with α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$cl(\{y_\sigma\})$ there exist disjoint Fα^m-OS $\mathcal{U}, \mathcal{V}$ such that α^m-$cl(\alpha^m$-$ker(\{x_\lambda\})) \leq \mathcal{U}$ and α^m-$cl(\alpha^m$-$ker(\{y_\sigma\})) \leq \mathcal{V}$. Since $\{x_\lambda\} \leq \alpha^m$-$ker(\{x_\lambda\})$, then α^m-$cl(\{x_\lambda\}) \leq \alpha^m$-$cl(\alpha^m$-$ker(\{x_\lambda\}))$ for each $x \in X$. So we get α^m-$cl(\{x_\lambda\}) \leq \mathcal{U}$ and α^m-$cl(\{y_\sigma\}) \leq \mathcal{V}$. Thus, (X,τ) is a Fα^m-R_1-space.

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Definition 4.12: Let (X, τ) be a fts. Then X is called:
(i) fuzzy α^m-regular space (F$\alpha^m r$-space, for short), if for each fuzzy point x_λ and each Fα^m-CS $\mathcal{F}$ such that $x_\lambda \in 1_X - \mathcal{F}$, there exist disjoint F$\alpha^m$-OS $\mathcal{U}$ and $\mathcal{V}$ such that $x_\lambda \in \mathcal{U}$ and $\mathcal{F} \leq \mathcal{V}$.
(ii) fuzzy α^m-normal space (F$\alpha^m n$-space, for short) iff for each pair of disjoint Fα^m-CS $\mathcal{A}$ and $\mathcal{B}$, there exist disjoint Fα^m-OS $\mathcal{U}$ and $\mathcal{V}$ such that $\mathcal{A} \leq \mathcal{U}$ and $\mathcal{B} \leq \mathcal{V}$.
(iii) fuzzy α^m-R_2-space (Fα^m-R_2-space, for short) if it is property F$\alpha^m r$-space.
(iv) fuzzy α^m-R_3-space (Fα^m-R_3-space, for short) iff it is Fα^m-R_1-space and F$\alpha^m n$-space.

Example 4.13: Consider the fts (X, τ) of example (3.2). Then (X, τ) is a F$\alpha^m r$-space and F$\alpha^m n$-space.

Remark 4.14: Every Fα^m-R_k-space is a Fα^m-R_{k-1}-space, $k = 2,3$.

Proof: Clearly.

Theorem 4.15: A fts (X, τ) is F$\alpha^m r$-space (Fα^m-R_2-space) iff for each Fα^m-closed subset $\mathcal{A}$ of X and $x_\lambda \notin \mathcal{A}$ with α^m-$ker(\mathcal{A}) \neq \alpha^m$-$ker(\{x_\lambda\})$ then there exist Fα^m-CS $\mathcal{F}_1$,$\mathcal{F}_2$ such that α^m-$ker(\mathcal{A}) \leq \mathcal{F}_1$, α^m-$ker(\mathcal{A}) \wedge \mathcal{F}_2 = 0_X$ and α^m-$ker(\{x_\lambda\}) \leq \mathcal{F}_2$, α^m-$ker(\{x_\lambda\}) \wedge \mathcal{F}_1 = 0_X$ and $\mathcal{F}_1 \vee \mathcal{F}_2 = 1_X$.

Proof: Let (X, τ) be a F$\alpha^m r$-space (Fα^m-R_2-space) and let $\mathcal{A}$ be a Fα^m-CS, $x_\lambda \notin \mathcal{A}$, then there exist disjoint Fα^m-OS $\mathcal{U}, \mathcal{V}$ such that $\mathcal{A} \leq \mathcal{U}$, $x_\lambda \in \mathcal{V}$ and $\mathcal{U} \wedge \mathcal{V} = 0_X$, then $\mathcal{U}^c$ and $\mathcal{V}^c$ are Fα^m-CS such that $\mathcal{U}^c \vee \mathcal{V}^c = 1_X$.
Put $\mathcal{F}_2 = \mathcal{U}^c$ and $\mathcal{F}_1 = \mathcal{V}^c$, so we get α^m-$ker(\mathcal{A}) \leq \mathcal{U} \leq \mathcal{F}_1$, α^m-$ker(\mathcal{A}) \wedge \mathcal{F}_2 = 0_X$ and α^m-$ker(\{x_\lambda\}) \leq \mathcal{V} \leq \mathcal{F}_2$, α^m-$ker(\{x_\lambda\}) \wedge \mathcal{F}_1 = 0_X$ and $\mathcal{F}_1 \vee \mathcal{F}_2 = 1_X$.
Conversely, let for each Fα^m-closed subset $\mathcal{A}$ of X and $x_\lambda \notin \mathcal{A}$ with α^m-$ker(\mathcal{A}) \neq \alpha^m$-$ker(\{x_\lambda\})$, then there exist F$\alpha^m$-CS $\mathcal{F}_1$,$\mathcal{F}_2$ such that α^m-$ker(\mathcal{A}) \leq \mathcal{F}_1$, α^m-$ker(\mathcal{A}) \wedge \mathcal{F}_2 = 0_X$ and α^m-$ker(\{x_\lambda\}) \leq \mathcal{F}_2$, α^m-$ker(\{x_\lambda\}) \wedge \mathcal{F}_1 = 0_X$ and $\mathcal{F}_1 \vee \mathcal{F}_2 = 1_X$. Then $\mathcal{F}_1^c$ and $\mathcal{F}_2^c$ are Fα^m-OS such that $\mathcal{F}_1^c \wedge \mathcal{F}_2^c = 0_X$ and α^m-$ker(\mathcal{A}) \wedge \mathcal{F}_1^c = 0_X$, α^m-$ker(\{x_\lambda\}) \wedge \mathcal{F}_2^c = 0_X$. So that $\mathcal{A} \leq \mathcal{F}_2^c$ and $x_\lambda \in \mathcal{F}_1^c$. Thus, (X, τ) is a F$\alpha^m r$-space (Fα^m-R_2-space).

Lemma 4.16: Let (X, τ) be a F$\alpha^m r$-space and $\mathcal{F}$ be a Fα^m-CS. Then α^m-$ker(\mathcal{F}) = \mathcal{F} = \alpha^m$-$cl(\mathcal{F})$.

Proof: Let (X, τ) be a F$\alpha^m r$-space and $\mathcal{F}$ be a Fα^m-CS. Then for each $x_\lambda \notin \mathcal{F}$, there exist disjoint F$\alpha^m$-OS $\mathcal{U}, \mathcal{V}$ such that $\mathcal{F} \leq \mathcal{U}$ and $x_\lambda \in \mathcal{V}$. Since α^m-$ker(\mathcal{F}) \leq \mathcal{U}$, implies α^m-$ker(\mathcal{F}) \wedge \mathcal{V} = 0_X$, thus $x_\lambda \notin \alpha^m$-$cl(\alpha^m$-$ker(\mathcal{F}))$. We showing that if $x_\lambda \notin \mathcal{F}$ implies $x_\lambda \notin \alpha^m$-$cl(\alpha^m$-$ker(\mathcal{F}))$, therefore α^m-$cl(\alpha^m$-$ker(\mathcal{F})) \leq \mathcal{F} = \alpha^m$-$cl(\mathcal{F})$. As α^m-$cl(\mathcal{F}) = \mathcal{F} \leq \alpha^m$-$ker(\mathcal{F})$ [by definition (4.1)]. Thus, α^m-$ker(\mathcal{F}) = \mathcal{F} = \alpha^m$-$cl(\mathcal{F})$.

Theorem 4.17: A fts (X, τ) is F$\alpha^m r$-space (Fα^m-R_2-space) iff for each Fα^m-closed subset $\mathcal{F}$ of X and $x_\lambda \notin \mathcal{F}$ with α^m-$cl(\alpha^m$-$ker(\mathcal{F})) \neq \alpha^m$-$cl(\alpha^m$-$ker(\{x_\lambda\}))$, then there exist disjoint Fα^m-OS $\mathcal{U}, \mathcal{V}$ such that α^m-$cl(\alpha^m$-$ker(\mathcal{F})) \leq \mathcal{U}$ and α^m-$cl(\alpha^m$-$ker(\{x_\lambda\})) \leq \mathcal{V}$.

Proof: Let (X, τ) be a F$\alpha^m r$-space (Fα^m-R_2-space) and let $\mathcal{F}$ be a Fα^m-CS, $x_\lambda \notin \mathcal{F}$. Then there exist disjoint Fα^m-OS $\mathcal{U}, \mathcal{V}$ such that $\mathcal{F} \leq \mathcal{U}$ and $x_\lambda \in \mathcal{V}$. By lemma (4.16), α^m-$cl(\alpha^m$-$ker(\mathcal{F})) = \alpha^m$-$cl(\mathcal{F}) = \mathcal{F}$, in the other hand (X, τ) is a Fα^m-R_0-space [by theorem (4.9) and remark (4.14)]. Hence, by theorem (4.6), α^m-$cl(\{x_\lambda\}) = \alpha^m$-$ker(\{x_\lambda\})$, for each $x \in X$. Thus, α^m-$cl(\alpha^m$-$ker(\mathcal{F})) \leq \mathcal{U}$ and α^m-$cl(\alpha^m$-$ker(\{x_\lambda\})) \leq \mathcal{V}$.
Conversely, let for each Fα^m-CS $\mathcal{F}$ and $x_\lambda \notin \mathcal{F}$ with α^m-$cl(\alpha^m$-$ker(\mathcal{F})) \neq \alpha^m$-$cl(\alpha^m$-$ker(\{x_\lambda\}))$, then there exist disjoint Fα^m-OS $\mathcal{U}, \mathcal{V}$ such that α^m-$cl(\alpha^m$-$ker(\mathcal{F})) \leq \mathcal{U}$ and α^m-$cl(\alpha^m$-$ker(\{x_\lambda\})) \leq \mathcal{V}$. Then $\mathcal{F} \leq \mathcal{U}$ and $x_\lambda \in \mathcal{V}$. Thus, (X, τ) is a F$\alpha^m r$-space (Fα^m-R_2-space).

Theorem 4.18: A fts (X, τ) is F$\alpha^m n$-space iff for each disjoint Fα^m-CS $\mathcal{A}$,$\mathcal{B}$ with α^m-$ker(\mathcal{A}) \neq \alpha^m$-$ker(\mathcal{B})$ then there exist Fα^m-CS $\mathcal{F}_1$,$\mathcal{F}_2$ such that α^m-$ker(\mathcal{A}) \leq \mathcal{F}_1$, α^m-$ker(\mathcal{A}) \wedge \mathcal{F}_2 = 0_X$ and α^m-$ker(\mathcal{B}) \leq \mathcal{F}_2$, α^m-$ker(\mathcal{B}) \wedge \mathcal{F}_1 = 0_X$ and $\mathcal{F}_1 \vee \mathcal{F}_2 = 1_X$.

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Proof: Let (X,τ) be a F$\alpha^m n$-space and let for each disjoint Fα^m-CS $\mathcal{A}$,$\mathcal{B}$ with α^m-$ker(\mathcal{A}) \neq \alpha^m$-$ker(\mathcal{B})$ then there exist disjoint Fα^m-OS $\mathcal{U},\mathcal{V}$ such that $\mathcal{A} \leq \mathcal{U}$ and $\mathcal{B} \leq \mathcal{V}$ and $\mathcal{U} \wedge \mathcal{V} = 0_X$, then $\mathcal{U}^c$ and $\mathcal{V}^c$ are Fα^m-CS such that $\mathcal{U}^c \vee \mathcal{V}^c = 1_X$ and α^m-$ker(\mathcal{A}) \wedge \mathcal{U}^c = 0_X$, α^m-$ker(\mathcal{B}) \wedge \mathcal{V}^c = 0_X$ Put $\mathcal{U}^c = \mathcal{F}_2$ and $\mathcal{V}^c = \mathcal{F}_1$. Thus, α^m-$ker(\mathcal{A}) \leq \mathcal{F}_1$, α^m-$ker(\mathcal{A}) \wedge \mathcal{F}_2 = 0_X$ and α^m-$ker(\mathcal{B}) \leq \mathcal{F}_2$, α^m-$ker(\mathcal{B}) \wedge \mathcal{F}_1 = 0_X$.
Conversely, let for each disjoint Fα^m-CS $\mathcal{A}$,$\mathcal{B}$ with α^m-$ker(\mathcal{A}) \neq \alpha^m$-$ker(\mathcal{B})$, there exist F$\alpha^m$-CS $\mathcal{F}_1$,$\mathcal{F}_2$ such that α^m-$ker(\mathcal{A}) \leq \mathcal{F}_1$, α^m-$ker(\mathcal{A}) \wedge \mathcal{F}_2 = 0_X$ and α^m-$ker(\mathcal{B}) \leq \mathcal{F}_2$, α^m-$ker(\mathcal{B}) \wedge \mathcal{F}_1 = 0_X$ and $\mathcal{F}_1 \vee \mathcal{F}_2 = 1_X$ implies $\mathcal{F}_1^c$ and $\mathcal{F}_2^c$ are Fα^m-OS such that $\mathcal{F}_1^c \wedge \mathcal{F}_2^c = 0_X$. Put $\mathcal{F}_1^c = \mathcal{V}$ and $\mathcal{F}_2^c = \mathcal{U}$, thus α^m-$ker(\mathcal{A}) \leq \mathcal{U}$ and α^m-$ker(\mathcal{B}) \leq \mathcal{V}$, so that $\mathcal{A} \leq \mathcal{U}$ and $\mathcal{B} \leq \mathcal{V}$. Thus (X,τ) is a F$\alpha^m n$-space.

Theorem 4.19: Every Fα^m-R_3-space is a F$\alpha^m r$-space.

Proof: Let $\mathcal{F}$ be a Fα^m-CS and $x_\lambda \notin \mathcal{F}$. Then α^m-$ker(\{x_\lambda\}) \neq \alpha^m$-$ker(\mathcal{F})$, then for each $y_\sigma \in \mathcal{F}$ there exist Fα^m-CS $\mathcal{A}_{y_\sigma}$,$\mathcal{B}_{y_\sigma}$ such that α^m-$ker(\{y_\sigma\}) \leq \mathcal{A}_{y_\sigma}$, α^m-$ker(\{y_\sigma\}) \wedge \mathcal{B}_{y_\sigma} = 0_X$ and α^m-$ker(\{x_\lambda\}) \leq \mathcal{B}_{y_\sigma}$, α^m-$ker(\{x_\lambda\}) \wedge \mathcal{A}_{y_\sigma} = 0_X$ [since (X,τ) is a Fα^m-R_1-space and by theorem (4.10)], let $\delta = \wedge\{\mathcal{B}_{y_\sigma}: x_\lambda \in \mathcal{B}_{y_\sigma}\}$, so we have $\delta \wedge \mathcal{F} = 0_X$. Hence (X,τ) is a F$\alpha^m n$-space, then there exist disjoint Fα^m-OS $\mathcal{U},\mathcal{V}$ such that $\mathcal{F} \leq \mathcal{U}$ and $x_\lambda \in \delta \leq \mathcal{V}$. Thus, (X,τ) is a F$\alpha^m r$-space.

5. FUZZY α^m-T_j-SPACES, $j = 0,1,2,3,4$

Definition 5.1: Let (X,τ) be a fts. Then X is called:
(i) fuzzy α^m-T_0-space (Fα^m-T_0-space, for short) iff for each pair of distinct fuzzy points in X, there exists a Fα^m-OS in X containing one and not the other.
(ii) fuzzy α^m-T_1-space (Fα^m-T_1-space, for short) iff for each pair of distinct fuzzy points x_λ and y_σ of X, there exists Fα^m-OS $\mathcal{U},\mathcal{V}$ containing x_λ and y_σ respectively such that $y_\sigma \notin \mathcal{U}$ and $x_\lambda \notin \mathcal{V}$.
(iii) fuzzy α^m-T_2-space (Fα^m-T_2-space, for short) iff for each pair of distinct fuzzy points x_λ and y_σ of X, there exist disjoint Fα^m-OS $\mathcal{U},\mathcal{V}$ in X such that $x_\lambda \in \mathcal{U}$ and $y_\sigma \in \mathcal{V}$.
(iv) fuzzy α^m-T_3-space (Fα^m-T_3-space, for short) iff it is Fα^m-T_1-space and F$\alpha^m r$-space.
(v) fuzzy α^m-T_4-space (Fα^m-T_4-space, for short) iff it is Fα^m-T_1-space and F$\alpha^m n$-space.

Example 5.2: Let $X = \{a,b\}$ and $\tau = \{0_X, a_1, 1_X\}$ be a fts on X. Then a_1 is a crisp point in X and (X,τ) is a Fα^m-T_0-space.

Example 5.3: Let $X = \{u,v\}$ and $\tau = \{0_X, u_1, v_1, 1_X\}$ be a fts on X. Then u_1, v_1 are crisp points in X and (X,τ) is a Fα^m-T_1-space and Fα^m-T_2-space.

Example 5.4: The discrete fuzzy topology in $X = [-2,2]$ is a Fα^m-T_3-space and Fα^m-T_4-space.

Remark 5.5: Every Fα^m-T_k-space is a Fα^m-T_{k-1}-space, $k = 1,2,3,4$.

Proof: Clearly.

Theorem 5.6: A fts (X,τ) is Fα^m-T_0-space iff either $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ or $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$, for each $x \neq y \in X$.

Proof: Let (X,τ) be a Fα^m-T_0-space then for each $x \neq y \in X$, there exists a Fα^m-OS $\mathcal{U}$ such that $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ or $x_\lambda \notin \mathcal{U}, y_\sigma \in \mathcal{U}$. Thus either $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ implies $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ or $x_\lambda \notin \mathcal{U}, y_\sigma \in \mathcal{U}$ implies $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$.
Conversely, let either $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ or $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$, for each $x \neq y \in X$. Then there exists a Fα^m-OS $\mathcal{U}$ such that $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ or $x_\lambda \notin \mathcal{U}, y_\sigma \in \mathcal{U}$. Thus (X,τ) is a Fα^m-T_0-space.

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Theorem 5.7: A fts (X,τ) is Fα^m-T_0-space iff either α^m-$ker(\{x_\lambda\})$ is weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$ or α^m-$ker(\{y_\sigma\})$ is weakly ultra fuzzy α^m-separated from $\{x_\lambda\}$ for each $x \neq y \in X$.

Proof: Let (X,τ) be a Fα^m-T_0-space then for each $x \neq y \in X$, there exists a Fα^m-OS $\mathcal{U}$ such that $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ or $x_\lambda \notin \mathcal{U}, y_\sigma \in \mathcal{U}$. Now if $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ implies α^m-$ker(\{x_\lambda\})$ is weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$. Or if $x_\lambda \notin \mathcal{U}, y_\sigma \in \mathcal{U}$ implies α^m-$ker(\{y_\sigma\})$ is weakly ultra fuzzy α^m-separated from $\{x_\lambda\}$.
Conversely, let either α^m-$ker(\{x_\lambda\})$ be weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$ or α^m-$ker(\{y_\sigma\})$ be weakly ultra fuzzy α^m-separated from $\{x_\lambda\}$. Then there exists a Fα^m-OS $\mathcal{U}$ such that α^m-$ker(\{x_\lambda\}) \leq \mathcal{U}$ and $y_\sigma \notin \mathcal{U}$ or α^m-$ker(\{y_\sigma\}) \leq \mathcal{U}, x_\lambda \notin \mathcal{U}$ implies $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ or $x_\lambda \notin \mathcal{U}, y_\sigma \in \mathcal{U}$. Thus, (X,τ) is a Fα^m-T_0-space.

Theorem 5.8: A fts (X,τ) is Fα^m-T_1-space iff for each $x \neq y \in X$, α^m-$ker(\{x_\lambda\})$ is weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$ and α^m-$ker(\{y_\sigma\})$ is weakly ultra fuzzy α^m-separated from $\{x_\lambda\}$.

Proof: Let (X,τ) be a Fα^m-T_1-space then for each $x \neq y \in X$, there exist Fα^m-OS $\mathcal{U},\mathcal{V}$ such that $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ and $x_\lambda \notin \mathcal{V}, y_\sigma \in \mathcal{V}$. Implies α^m-$ker(\{x_\lambda\})$ is weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$ and α^m-$ker(\{y_\sigma\})$ is weakly ultra fuzzy α^m-separated from $\{x_\lambda\}$.
Conversely, let α^m-$ker(\{x_\lambda\})$ be weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$ and α^m-$ker(\{y_\sigma\})$ be weakly ultra fuzzy α^m-separated from $\{x_\lambda\}$. Then there exist Fα^m-OS $\mathcal{U},\mathcal{V}$ such that α^m-$ker(\{x_\lambda\}) \leq \mathcal{U}, y_\sigma \notin \mathcal{U}$ and α^m-$ker(\{y_\sigma\}) \leq \mathcal{V}, x_\lambda \notin \mathcal{V}$ implies $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ and $x_\lambda \notin \mathcal{V}, y_\sigma \in \mathcal{V}$. Thus, (X,τ) is a Fα^m-T_1-space.

Theorem 5.9: A fts (X,τ) is Fα^m-T_1-space iff for each $x \in X$, α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$.

Proof: Let (X,τ) be a Fα^m-T_1-space and let α^m-$ker(\{x_\lambda\}) \neq \{x_\lambda\}$. Then α^m-$ker(\{x_\lambda\})$ contains another fuzzy point distinct from x_λ say y_σ. So $y_\sigma \in \alpha^m$-$ker(\{x_\lambda\})$ implies α^m-$ker(\{x_\lambda\})$ is not weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$. Hence by theorem (5.8), (X,τ) is not a Fα^m-T_1-space this is contradiction. Thus α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$.
Conversely, let α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$, for each $x \in X$ and let (X,τ) be not a Fα^m-T_1-space. Then by theorem (5.8), α^m-$ker(\{x_\lambda\})$ is not weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$, this means that for every Fα^m-OS $\mathcal{U}$ contains α^m-$ker(\{x_\lambda\})$ then $y_\sigma \in \mathcal{U}$ implies $y_\sigma \in \wedge\{\mathcal{U} \in \mathrm{F}\alpha^m\text{-}O(X): x_\lambda \in \mathcal{U}\}$ implies $y_\sigma \in \alpha^m$-$ker(\{x_\lambda\})$, this is contradiction. Thus, (X,τ) is a Fα^m-T_1-space.

Theorem 5.10: A fts (X,τ) is Fα^m-T_1-space iff for each $x \neq y \in X$, $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ and $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$.

Proof: Let (X,τ) be a Fα^m-T_1-space then for each $x \neq y \in X$, there exists Fα^m-OS $\mathcal{U},\mathcal{V}$ such that $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ and $y_\sigma \in \mathcal{V}, x_\lambda \notin \mathcal{U}$. Implies $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ and $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$.
Conversely, let $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ and $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$, for each $x \neq y \in X$. Then there exists Fα^m-OS $\mathcal{U},\mathcal{V}$ such that $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ and $y_\sigma \in \mathcal{V}, x_\lambda \notin \mathcal{V}$. Thus, (X,τ) is a Fα^m-T_1-space.

Theorem 5.11: A fts (X,τ) is Fα^m-T_1-space iff for each $x \neq y \in X$ implies α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$.

Proof: Let (X,τ) be a Fα^m-T_1-space. Then α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$ and α^m-$ker(\{y_\sigma\}) = \{y_\sigma\}$ [by theorem (5.9)]. Thus, α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$.
Conversely, let for each $x \neq y \in X$ implies α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$ and let (X,τ) be not Fα^m-T_1-space then for each $x \neq y \in X$ implies $y_\sigma \in \alpha^m$-$ker(\{x_\lambda\})$ or $x_\lambda \in \alpha^m$-$ker(\{y_\sigma\})$ [by theorem (5.10)], then α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) \neq 0_X$ this is contradiction. Thus, (X,τ) is a Fα^m-T_1-space.

Theorem 5.12: A fts (X,τ) is Fα^m-T_1-space iff (X,τ) is Fα^m-T_0-space and Fα^m-R_0-space.

Proof: Let (X,τ) be a Fα^m-T_1-space and let $x_\lambda \in \mathcal{U}$ be a Fα^m-OS, then for each $x \neq y \in X$, α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$ [by theorem (5.11)] implies $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ and $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ this means α^m-$cl(\{x_\lambda\}) = \{x_\lambda\}$, hence α^m-$cl(\{x_\lambda\}) \leq \mathcal{U}$. Thus, (X,τ) is a Fα^m-R_0-space.

Conversely, let (X,τ) be a Fα^m-T_0-space and Fα^m-R_0-space, then for each $x \neq y \in X$ there exists a Fα^m-OS $\mathcal{U}$ such that $x_\lambda \in \mathcal{U}$, $y_\sigma \notin \mathcal{U}$ or $x_\lambda \notin \mathcal{U}, y_\sigma \in \mathcal{U}$. Say $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ since (X,τ) is a Fα^m-R_0-space, then α^m-$cl(\{x_\lambda\}) \leq \mathcal{U}$, this means there exists a Fα^m-OS $\mathcal{V}$ such that $y_\sigma \in \mathcal{V}, x_\lambda \notin \mathcal{V}$. Thus, (X,τ) is a Fα^m-T_1-space.

Theorem 5.13: A fts (X,τ) is Fα^m-T_2-space iff
(i) (X,τ) is Fα^m-T_0-space and Fα^m-R_1-space.
(ii) (X,τ) is Fα^m-T_1-space and Fα^m-R_1-space.

Proof: (i) Let (X,τ) be a Fα^m-T_2-space then it is a Fα^m-T_0-space. Now since (X,τ) is a Fα^m-T_2-space then for each $x \neq y \in X$, there exist disjoint Fα^m-OS $\mathcal{U},\mathcal{V}$ such that $x_\lambda \in \mathcal{U}$ and $y_\sigma \in \mathcal{V}$ implies $x_\lambda \notin \alpha^m$-$cl(\{y_\sigma\})$ and $y_\alpha \notin \alpha^m$-$cl(\{x_\lambda\})$, therefore α^m-$cl(\{x_\lambda\}) = \{x_\lambda\} \leq \mathcal{U}$ and α^m-$cl(\{y_\sigma\}) = \{y_\sigma\} \leq \mathcal{V}$. Thus, (X,τ) is a Fα^m-R_1-space.
Conversely, let (X,τ) be a Fα^m-T_0-space and Fα^m-R_1-space, then for each $x \neq y \in X$, there exists a Fα^m-OS $\mathcal{U}$ such that $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ or $y_\sigma \in \mathcal{U}, x_\lambda \notin \mathcal{U}$, implies α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$cl(\{y_\sigma\})$, since (X,τ) is a Fα^m-R_1-space [by assumption], then there exist disjoint Fα^m-OS $\mathcal{M},\mathcal{N}$ such that $x_\lambda \in \mathcal{M}$ and $y_\sigma \in \mathcal{N}$. Thus, (X,τ) is a Fα^m-T_2-space.
(ii) By the same way of part (i) a Fα^m-T_2-space is Fα^m-T_1-space and Fα^m-R_1-space.
Conversely, let (X,τ) be a Fα^m-T_1-space and Fα^m-R_1-space, then for each $x \neq y \in X$, there exist Fα^m-OS $\mathcal{U},\mathcal{V}$ such that $x_\lambda \in \mathcal{U}, y_\sigma \notin \mathcal{U}$ and $y_\sigma \in \mathcal{V}, x_\lambda \notin \mathcal{V}$ implies α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$cl(\{y_\sigma\})$, since (X,τ) is a Fα^m-R_1-space, then there exist disjoint Fα^m-OS $\mathcal{M},\mathcal{N}$ such that $x_\lambda \in \mathcal{M}$ and $y_\sigma \in \mathcal{N}$. Thus, (X,τ) is a Fα^m-T_2-space.

Corollary 5.14: A Fα^m-T_0-space is Fα^m-T_2-space iff for each $x \neq y \in X$ with α^m-$ker(\{x_\lambda\}) \neq \alpha^m$-$ker(\{y_\sigma\})$ then there exist Fα^m-CS $\mathcal{A}_1$,$\mathcal{A}_2$ such that α^m-$ker(\{x_\lambda\}) \leq \mathcal{A}_1$, α^m-$ker(\{x_\lambda\}) \wedge \mathcal{A}_2 = 0_X$ and α^m-$ker(\{y_\sigma\}) \leq \mathcal{A}_2$, α^m-$ker(\{y_\sigma\}) \wedge \mathcal{A}_1 = 0_X$ and $\mathcal{A}_1 \vee \mathcal{A}_2 = 1_X$.

Proof: By theorem (4.10) and theorem (5.13).

Corollary 5.15: A Fα^m-T_1-space is Fα^m-T_2-space iff one of the following conditions holds:
(i) for each $x \neq y \in X$ with α^m-$cl(\{x_\lambda\}) \neq \alpha^m$-$cl(\{y_\sigma\})$, then there exist F$\alpha^m$-OS $\mathcal{U},\mathcal{V}$ such that α^m-$cl(\alpha^m$-$ker(\{x_\lambda\})) \leq \mathcal{U}$ and α^m-$cl(\alpha^m$-$ker(\{y_\sigma\})) \leq \mathcal{V}$.
(ii) for each $x \neq y \in X$ with α^m-$ker(\{x_\lambda\}) \neq \alpha^m$-$ker(\{y_\sigma\})$, then there exist F$\alpha^m$-CS $\mathcal{A}_1$,$\mathcal{A}_2$ such that α^m-$ker(\{x_\lambda\}) \leq \mathcal{A}_1$, α^m-$ker(\{x_\lambda\}) \wedge \mathcal{A}_2 = 0_X$ and α^m-$ker(\{y_\sigma\}) \leq \mathcal{A}_2$, α^m-$ker(\{y_\sigma\}) \wedge \mathcal{A}_1 = 0_X$ and $\mathcal{A}_1 \vee \mathcal{A}_2 = 1_X$.

Proof: (i) By corollary (4.11) and theorem (5.13).
(ii) By theorem (4.10) and theorem (5.13).

Theorem 5.16: A Fα^m-R_1-space is Fα^m-T_2-space iff one of the following conditions holds:
(i) for each $x \in X$, α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$.
(ii) for each $x \neq y \in X$, α^m-$ker(\{x_\lambda\}) \neq \alpha^m$-$ker(\{y_\sigma\})$ implies α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$.
(iii) for each $x \neq y \in X$, either $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ or $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$.
(iv) for each $x \neq y \in X$, then $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ and $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$.

Proof: (i) Let (X,τ) be a Fα^m-T_2-space. Then (X,τ) is a Fα^m-T_1-space and Fα^m-R_1-space [by theorem (5.13)]. Hence by theorem (5.9), α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$ for each $x \in X$.
Conversely, let for each $x \in X$, α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$, then by theorem (5.9), (X,τ) is a Fα^m-T_1-space. Also (X,τ) is a Fα^m-R_1-space by assumption. Hence by theorem (5.13), (X,τ) is a Fα^m-T_2-space.
(ii) Let (X,τ) be a Fα^m-T_2-space. Then (X,τ) is Fα^m-T_1-space [by remark (5.5)]. Hence by theorem (5.11), α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$ for each $x \neq y \in X$.
Conversely, assume that for each $x \neq y \in X$, α^m-$ker(\{x_\lambda\}) \neq \alpha^m$-$ker(\{y_\sigma\})$ implies α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$. So by theorem (5.11), (X,τ) is a Fα^m-T_1-space, also (X,τ) is a Fα^m-R_1-space by assumption. Hence by theorem (5.13), (X,τ) is a Fα^m-T_2-space.

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(iii) Let (X,τ) be a Fα^m-T_2-space. Then (X,τ) is a Fα^m-T_0-space [by remark (5.5)]. Hence by theorem (5.6), either $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ or $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ for each $x \neq y \in X$.
Conversely, assume that for each $x \neq y \in X$, either $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ or $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$ for each $x \neq y \in X$. So by theorem (5.6), (X,τ) is a Fα^m-T_0-space, also (X,τ) is Fα^m-R_1-space by assumption. Thus (X,τ) is a Fα^m-T_2-space [by theorem (5.13)].
(iv) Let (X,τ) be a Fα^m-T_2-space. Then (X,τ) is a Fα^m-T_1-space and Fα^m-R_1-space [by theorem (5.13)]. Hence by theorem (5.10), $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ and $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$.
Conversely, let for each $x \neq y \in X$ then $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ and $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$. Then by theorem (5.10), (X,τ) is a Fα^m-T_1-space. Also (X,τ) is a Fα^m-R_1-space by assumption. Hence by theorem (5.13), (X,τ) is a Fα^m-T_2-space.

Remark 5.17: Each fuzzy separation axiom is defined as the conjunction of two weaker axioms: Fα^m-T_k-space = Fα^m-R_{k-1}-space and Fα^m-T_{k-1}-space = Fα^m-R_{k-1}-space and Fα^m-T_0-space, $k = 1,2,3,4$.

Theorem 5.18: Let (X,τ) be a fts and α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$ for each $x \in X$ then (X,τ) is Fα^m-T_3-space if and only if it is a Fα^m-R_2-space.

Proof: Let (X,τ) be a Fα^m-T_3-space. Then, (X,τ) is a Fα^m-R_2-space [By remark (5.17)].
Conversely, let (X,τ) be a Fα^m-R_2-space then it is a F$\alpha^m r$-space [definition (4.12)(iii)]. By assumption, α^m-$ker(\{x_\lambda\}) = \{x_\lambda\}$ for each $x \in X$, then (X,τ) is a Fα^m-T_1-space [by theorem (5.9)]. Hence by remark (5.17), (X,τ) is a Fα^m-T_3-space.

Theorem 5.19: Let (X,τ) be a fts and let $x \neq y \in X$, implies α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$, then (X,τ) is a Fα^m-T_3-space iff it is a Fα^m-R_2-space.

Proof: Let (X,τ) be a Fα^m-T_3-space. Then (X,τ) is a Fα^m-R_2-space [by remark (5.17)].
Conversely, let (X,τ) be a Fα^m-R_2-space then it is a F$\alpha^m r$-space [definition (4.12)(iii)]. By assumption, α^m-$ker(\{x_\lambda\}) \wedge \alpha^m$-$ker(\{y_\sigma\}) = 0_X$, for each $x \neq y \in X$, then by theorem (5.11), (X,τ) is a Fα^m-T_1-space. Hence by remark (5.17), (X,τ) is a Fα^m-T_3-space.

Theorem 5.20: Let (X,τ) be a fts and for each $x \neq y \in X$ either $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ or $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$, then (X,τ) is a Fα^m-T_3-space iff it is a Fα^m-R_2-space.

Proof: Let (X,τ) be a Fα^m-T_3-space. Then (X,τ) is a Fα^m-R_2-space [by remark (5.17)].
Conversely, let (X,τ) be a Fα^m-R_2-space then it is a F$\alpha^m r$-space [definition (4.12)(iii)]. By assumption, for each $x \neq y \in X$ either $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ or $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$. This means either α^m-$ker(\{x_\lambda\})$ is weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$ or α^m-$ker(\{y_\sigma\})$ is weakly ultra fuzzy α^m-separated from $\{x_\lambda\}$, so by theorem (5.7), (X,τ) is a Fα^m-T_0-space. Hence by remark (5.17), (X,τ) is a Fα^m-T_3-space.

Theorem 5.21: Let (X,τ) be a fts and let $x \neq y \in X$, then $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ and $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$, (X,τ) is a Fα^m-T_3-space iff it is a Fα^m-R_2-space.

Proof: Let (X,τ) be a Fα^m-T_3-space. Then (X,τ) is a Fα^m-R_2-space [by remark (5.17)].
Conversely, let (X,τ) be a Fα^m-R_2-space then it is a F$\alpha^m r$-space [definition (4.12)(iii)]. By assumption, for each $x \neq y \in X$ then $x_\lambda \notin \alpha^m$-$ker(\{y_\sigma\})$ and $y_\sigma \notin \alpha^m$-$ker(\{x_\lambda\})$. Therefore, α^m-$ker(\{x_\lambda\})$ is weakly ultra fuzzy α^m-separated from $\{y_\sigma\}$ and α^m-$ker(\{y_\sigma\})$ is weakly ultra fuzzy α^m-separated from $\{x_\lambda\}$, so by theorem (5.8), (X,τ) is a Fα^m-T_1-space. Hence by remark (5.17), (X,τ) is a Fα^m-T_3-space.

Remark 5.22: The relation between fuzzy α^m-separation axioms can be representing as a matrix. Therefore, the element a_{ij} refers to this relation. As the following matrix representation shows:

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and	$F\alpha^m$-T_0	$F\alpha^m$-T_1	$F\alpha^m$-T_2	$F\alpha^m$-T_3	$F\alpha^m$-T_4	$F\alpha^m$-R_0	$F\alpha^m$-R_1	$F\alpha^m$-R_2	$F\alpha^m$-R_3
$F\alpha^m$-T_0	$F\alpha^m$-T_0	$F\alpha^m$-T_1	$F\alpha^m$-T_2	$F\alpha^m$-T_3	$F\alpha^m$-T_4	$F\alpha^m$-T_1	$F\alpha^m$-T_2	$F\alpha^m$-R_3	$F\alpha^m$-T_4
$F\alpha^m$-T_1	$F\alpha^m$-T_1	$F\alpha^m$-T_1	$F\alpha^m$-T_2	$F\alpha^m$-T_3	$F\alpha^m$-T_4	$F\alpha^m$-T_1	$F\alpha^m$-T_2	$F\alpha^m$-R_3	$F\alpha^m$-T_4
$F\alpha^m$-T_2	$F\alpha^m$-T_2	$F\alpha^m$-T_2	$F\alpha^m$-T_2	$F\alpha^m$-T_3	$F\alpha^m$-T_4	$F\alpha^m$-T_2	$F\alpha^m$-T_2	$F\alpha^m$-R_3	$F\alpha^m$-T_4
$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_4	$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_4
$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4
$F\alpha^m$-R_0	$F\alpha^m$-T_1	$F\alpha^m$-T_1	$F\alpha^m$-T_2	$F\alpha^m$-T_3	$F\alpha^m$-T_4	$F\alpha^m$-R_0	$F\alpha^m$-R_1	$F\alpha^m$-R_2	$F\alpha^m$-R_3
$F\alpha^m$-R_1	$F\alpha^m$-T_2	$F\alpha^m$-T_2	$F\alpha^m$-T_2	$F\alpha^m$-T_3	$F\alpha^m$-T_4	$F\alpha^m$-R_1	$F\alpha^m$-R_1	$F\alpha^m$-R_2	$F\alpha^m$-R_3
$F\alpha^m$-R_2	$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_3	$F\alpha^m$-T_4	$F\alpha^m$-R_2	$F\alpha^m$-R_2	$F\alpha^m$-R_2	$F\alpha^m$-R_3
$F\alpha^m$-R_3	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-T_4	$F\alpha^m$-R_3	$F\alpha^m$-R_3	$F\alpha^m$-R_3	$F\alpha^m$-R_3

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